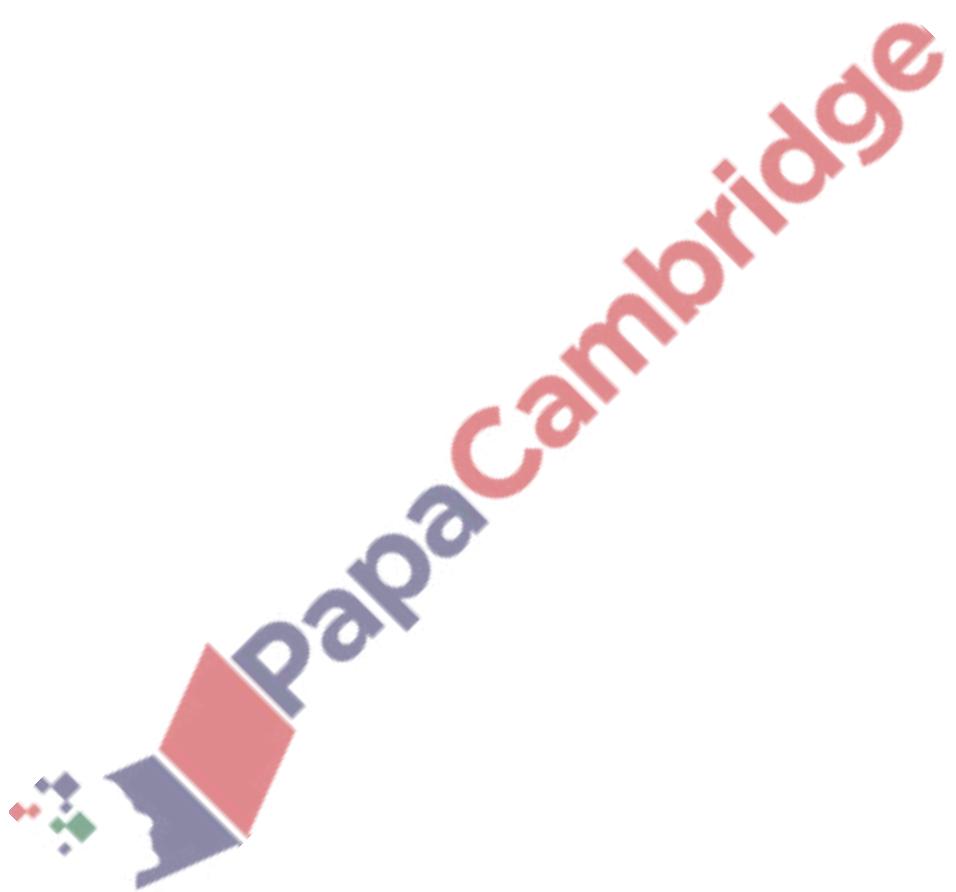

AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

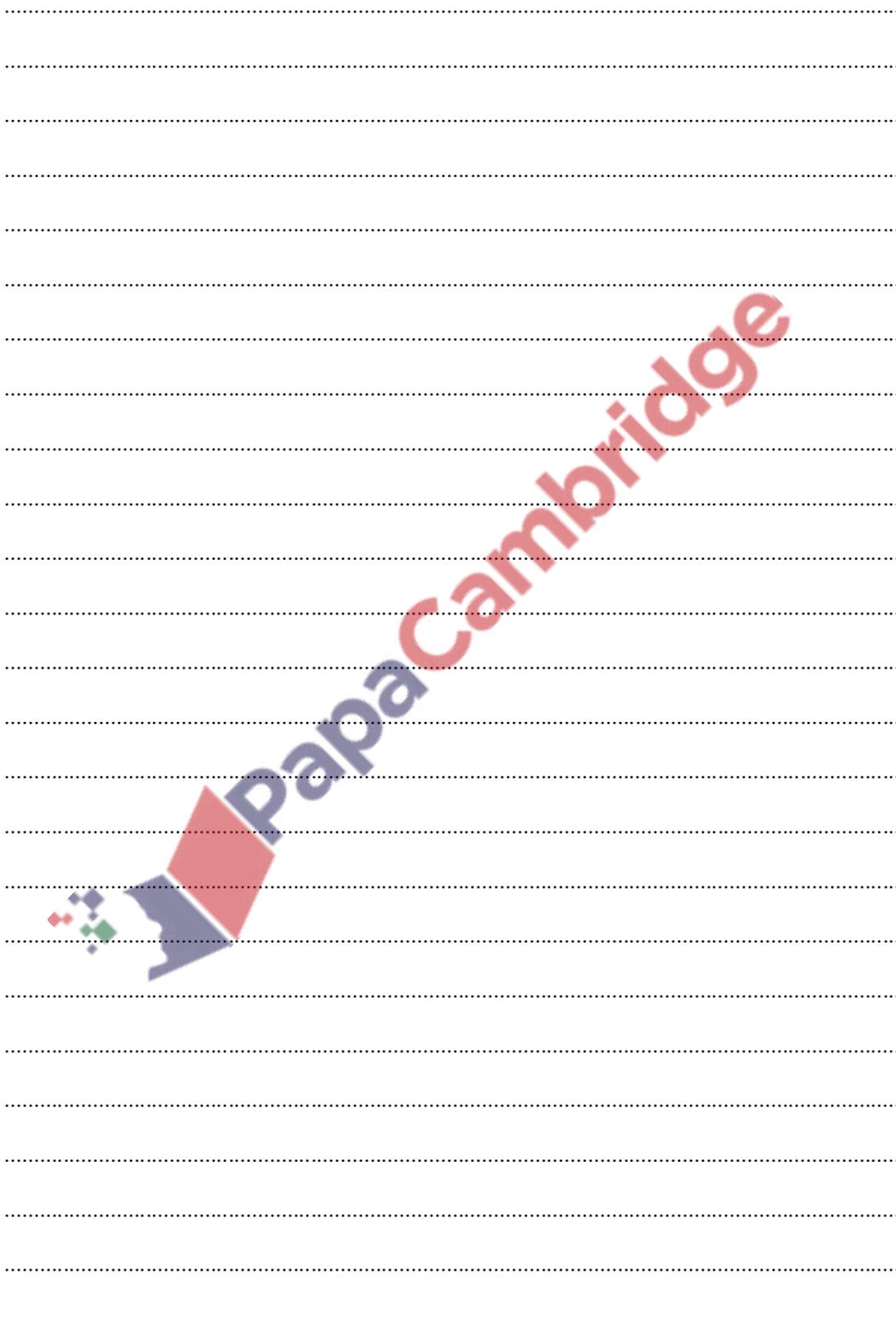
Chapter 5

Trigonometry

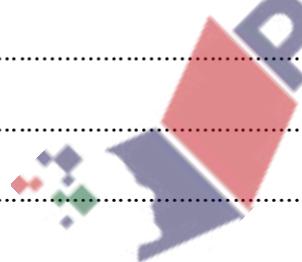


180. 9709_m22_qp_12_Q: 7

(a) Show that $\frac{\sin \theta + 2 \cos \theta}{\cos \theta - 2 \sin \theta} - \frac{\sin \theta - 2 \cos \theta}{\cos \theta + 2 \sin \theta} \equiv \frac{4}{5 \cos^2 \theta - 4}$. [4]



- (b) Hence solve the equation $\frac{\sin \theta + 2 \cos \theta}{\cos \theta - 2 \sin \theta} - \frac{\sin \theta - 2 \cos \theta}{\cos \theta + 2 \sin \theta} = 5$ for $0^\circ < \theta < 180^\circ$. [3]

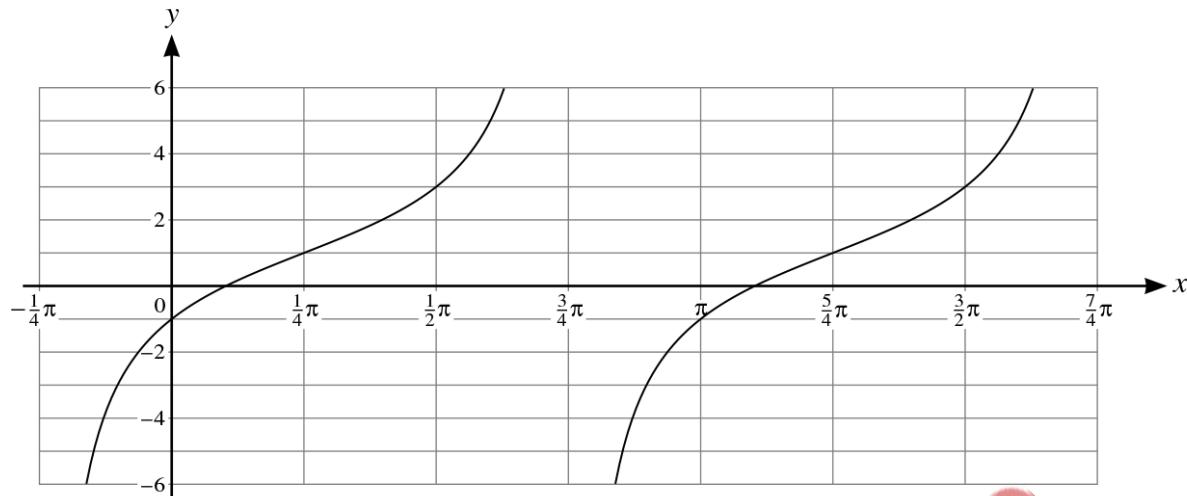


181. 9709_m21_qp_12 Q: 3

Solve the equation $\frac{\tan \theta + 2 \sin \theta}{\tan \theta - 2 \sin \theta} = 3$ for $0^\circ < \theta < 180^\circ$. [4]



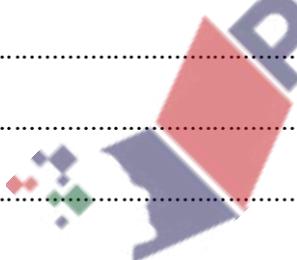
182. 9709_s21_qp_11 Q: 4



The diagram shows part of the graph of $y = a \tan(x - b) + c$.

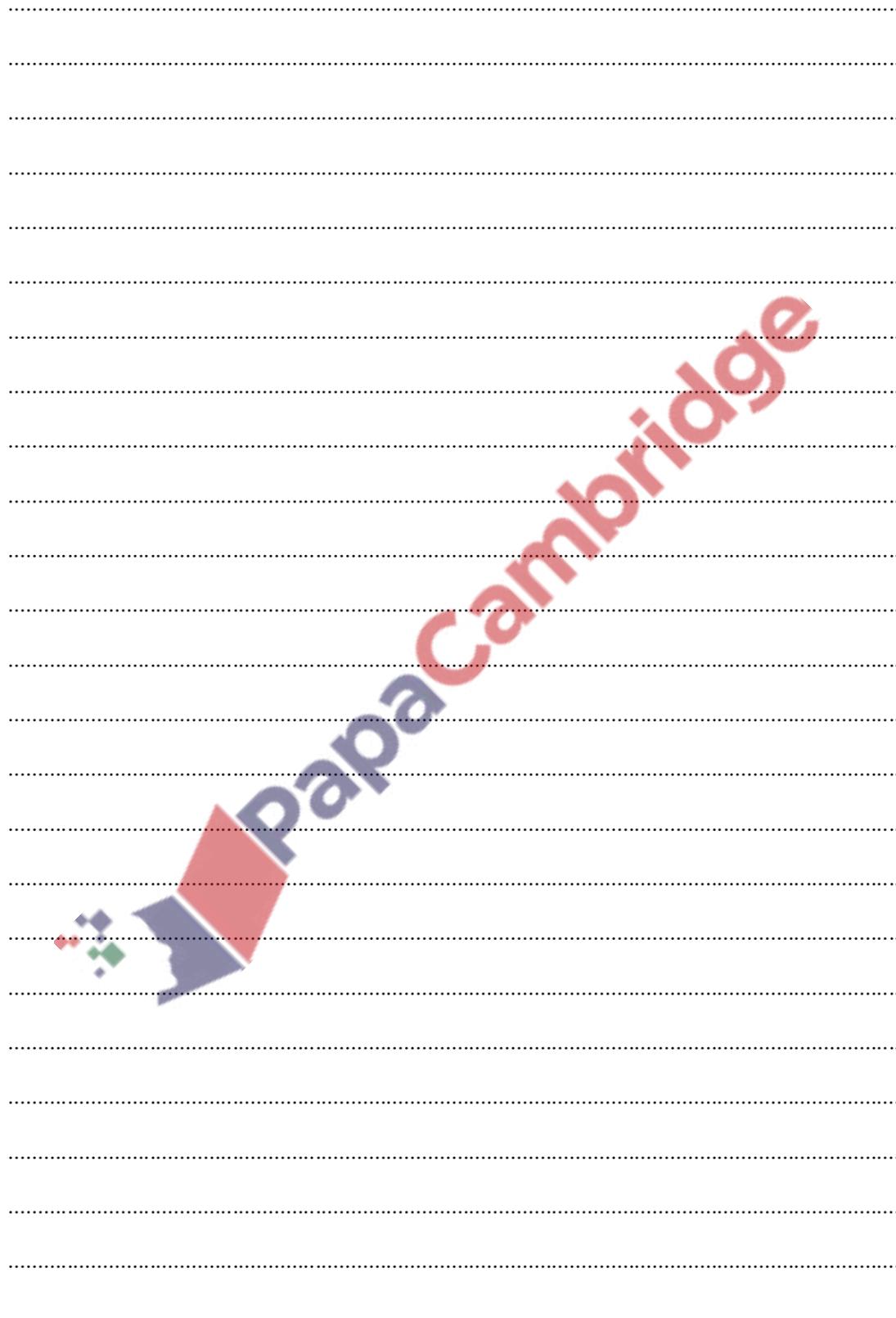
Given that $0 < b < \pi$, state the values of the constants a , b and c .

[3]

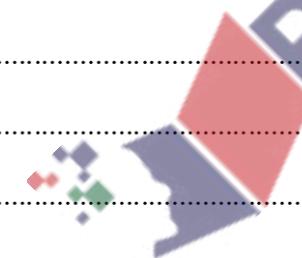


183. 9709_s21_qp_11 Q: 7

- (a) Prove the identity $\frac{1 - 2 \sin^2 \theta}{1 - \sin^2 \theta} \equiv 1 - \tan^2 \theta$. [2]

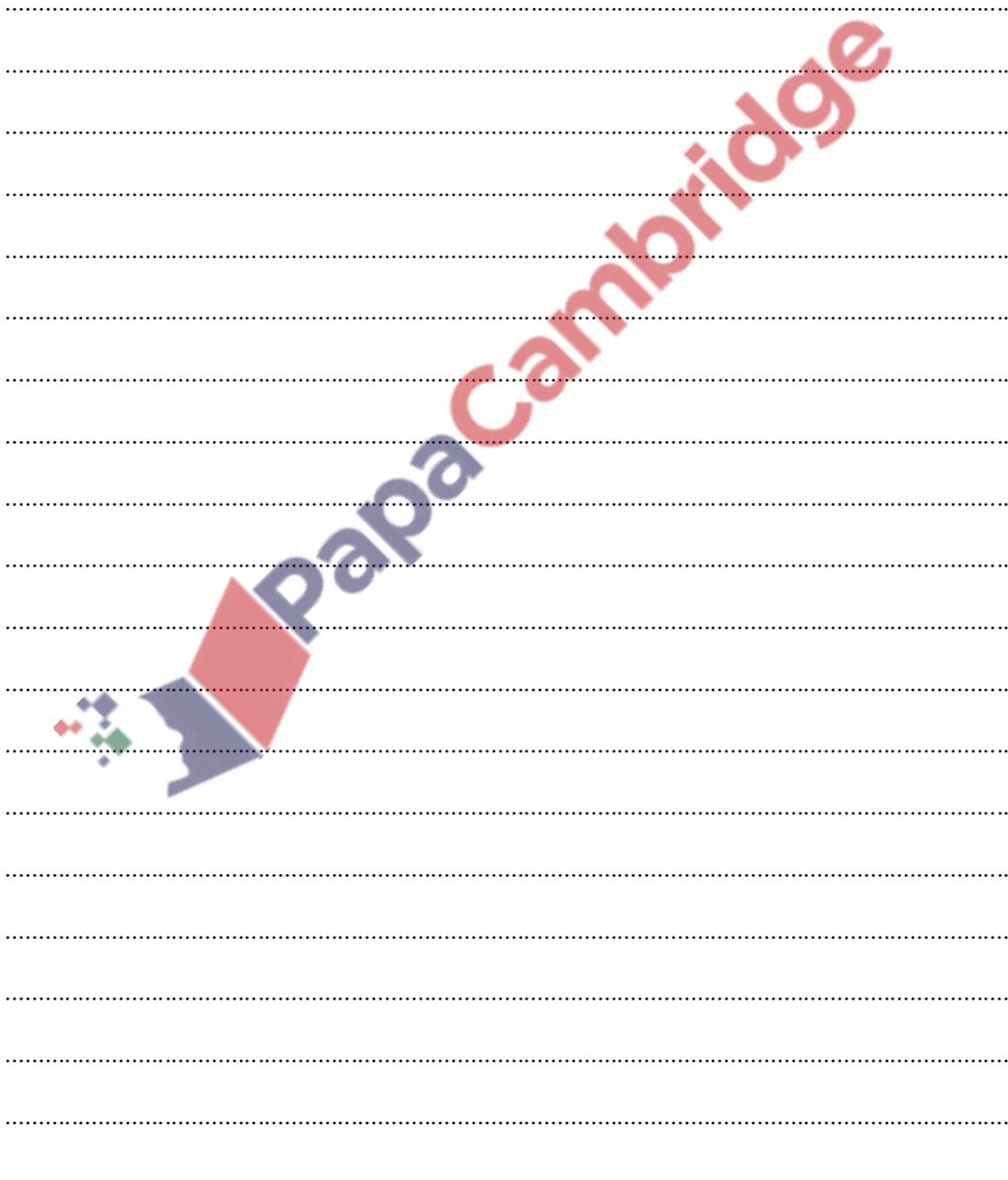


- (b) Hence solve the equation $\frac{1 - 2 \sin^2 \theta}{1 - \sin^2 \theta} = 2 \tan^4 \theta$ for $0^\circ \leq \theta \leq 180^\circ$. [3]

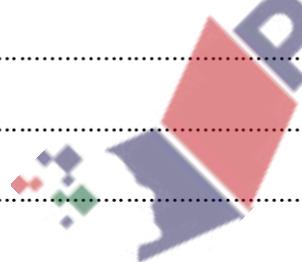


184. 9709_s21_qp_12 Q: 10

- (a) Prove the identity $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} \equiv \frac{4 \tan x}{\cos x}$. [4]



- (b) Hence solve the equation $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 8 \tan x$ for $0 \leq x \leq \frac{1}{2}\pi$. [3]



185. 9709_s21_qp_13 Q: 4

- (a) Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k. \quad [2]$$

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- (b) Hence express
- $\cos x$
- in terms of
- k
- .

[2]



- (c) Hence solve the equation
- $\frac{\tan x + \sin x}{\tan x - \sin x} = 4$
- for
- $-\pi < x < \pi$
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[2]

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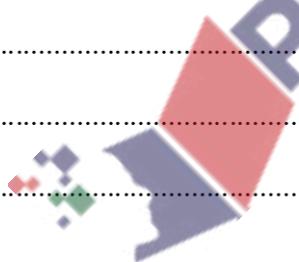
186. 9709_w21_qp_11 Q: 3

Solve, by factorising, the equation

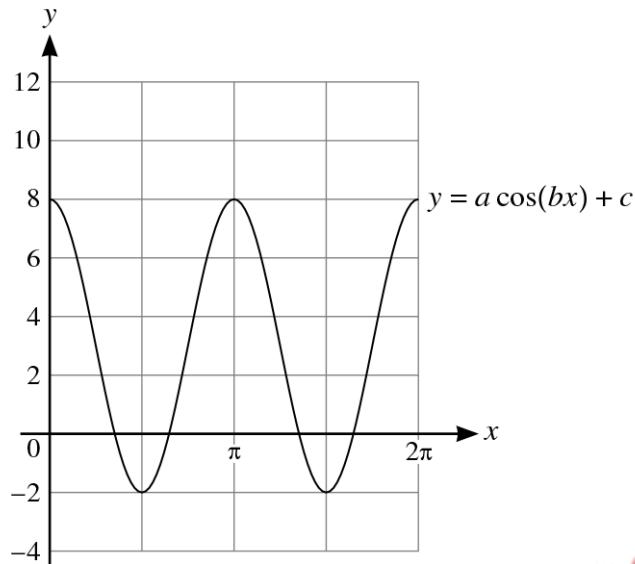
$$6 \cos \theta \tan \theta - 3 \cos \theta + 4 \tan \theta - 2 = 0,$$

for $0^\circ \leq \theta \leq 180^\circ$.

[4]



187. 9709_w21_qp_11 Q: 5



The diagram shows part of the graph of $y = a \cos(bx) + c$.

- (a) Find the values of the positive integers a , b and c . [3]

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- (b) For these values of a , b and c , use the given diagram to determine the number of solutions in the interval $0 \leq x \leq 2\pi$ for each of the following equations.

(i) $a \cos(bx) + c = \frac{6}{\pi}x$ [1]

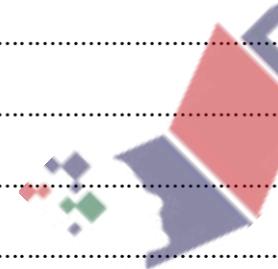
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(ii) $a \cos(bx) + c = 6 - \frac{6}{\pi}x$ [1]

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188. 9709_w21_qp_12 Q: 1

Solve the equation $2 \cos \theta = 7 - \frac{3}{\cos \theta}$ for $-90^\circ < \theta < 90^\circ$. [4]

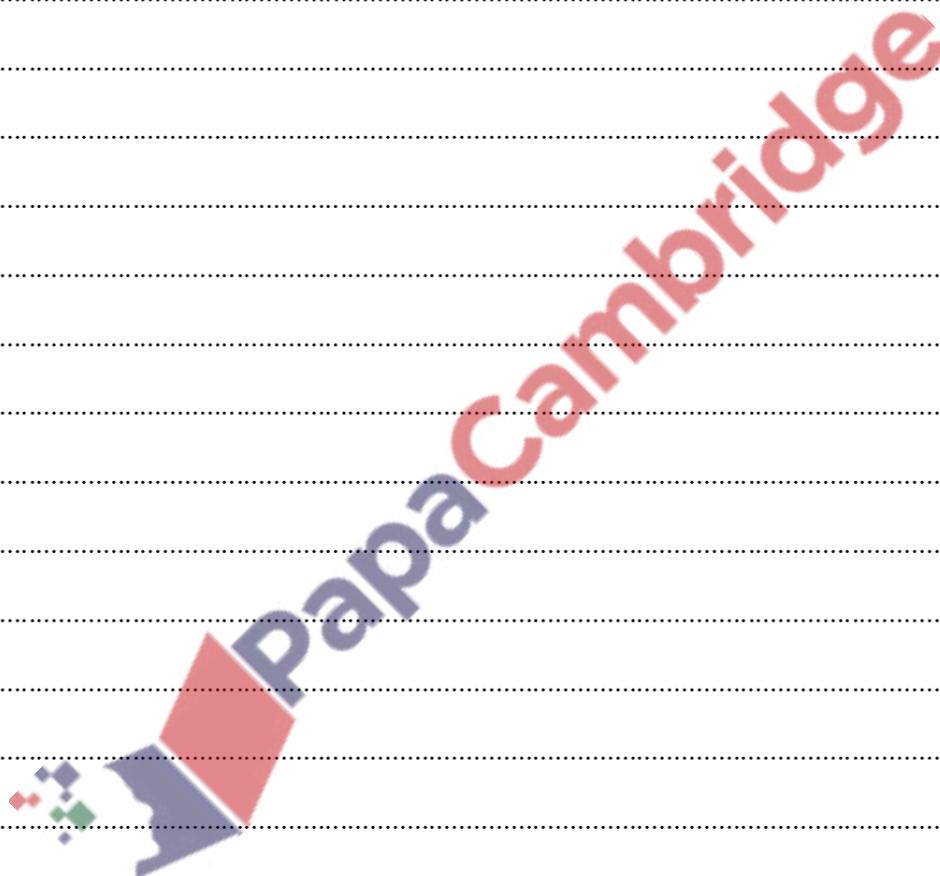


189. 9709_w21_qp_13 Q: 7

- (a) Show that the equation $\frac{\tan x + \cos x}{\tan x - \cos x} = k$, where k is a constant, can be expressed as

$$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0.$$

[4]



- (b) Hence solve the equation $\frac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0^\circ \leq x \leq 360^\circ$. [4]

190. 9709_m20_qp_12 Q: 5

Solve the equation

$$\frac{\tan \theta + 3 \sin \theta + 2}{\tan \theta - 3 \sin \theta + 1} = 2$$

for $0^\circ \leq \theta \leq 90^\circ$.

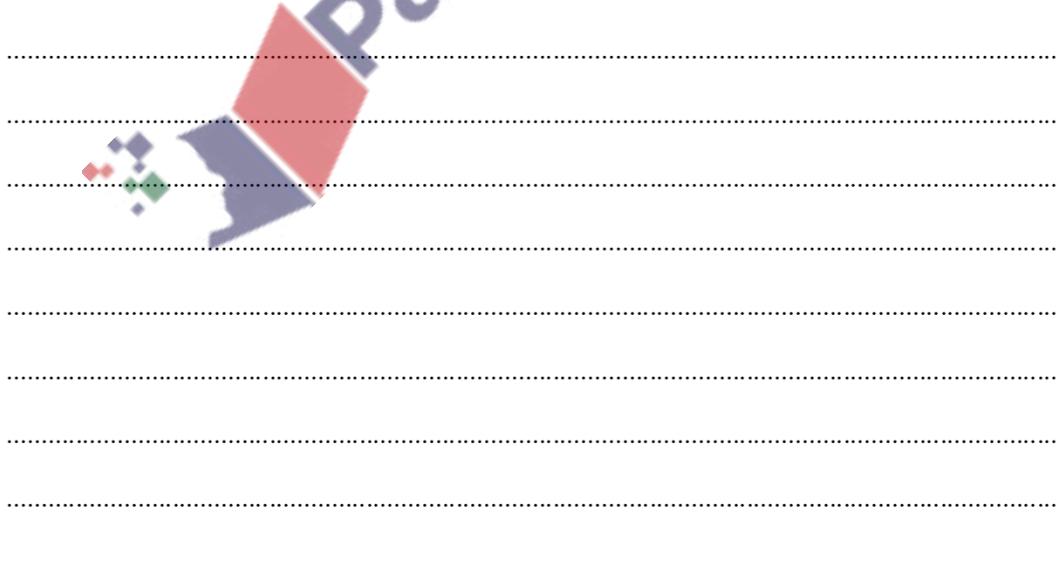
[5]



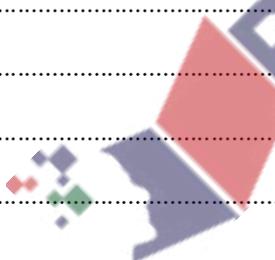
191. 9709_m20_qp_12 Q: 11

- (a) Solve the equation $3 \tan^2 x - 5 \tan x - 2 = 0$ for $0^\circ \leq x \leq 180^\circ$. [4]

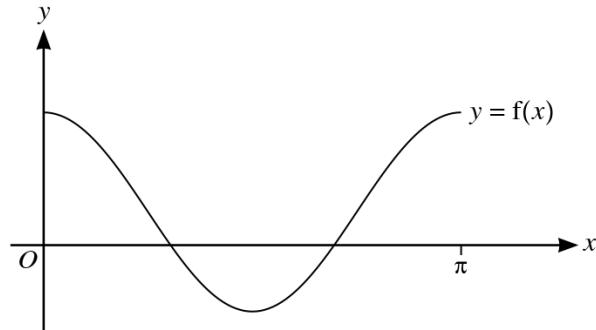
- (b) Find the set of values of k for which the equation $3 \tan^2 x - 5 \tan x + k = 0$ has no solutions. [2]



- (c) For the equation $3 \tan^2 x - 5 \tan x + k = 0$, state the value of k for which there are three solutions in the interval $0^\circ \leq x \leq 180^\circ$, and find these solutions. [3]



192. 9709_s20_qp_11 Q: 4



The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2} \cos 2x + \frac{1}{2}$ for $0 \leq x \leq \pi$.

- (a) State the range of f . [2]

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.....
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A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x -axis is a tangent to the curve $y = g(x)$.

- (b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ onto $y = g(x)$. [2]

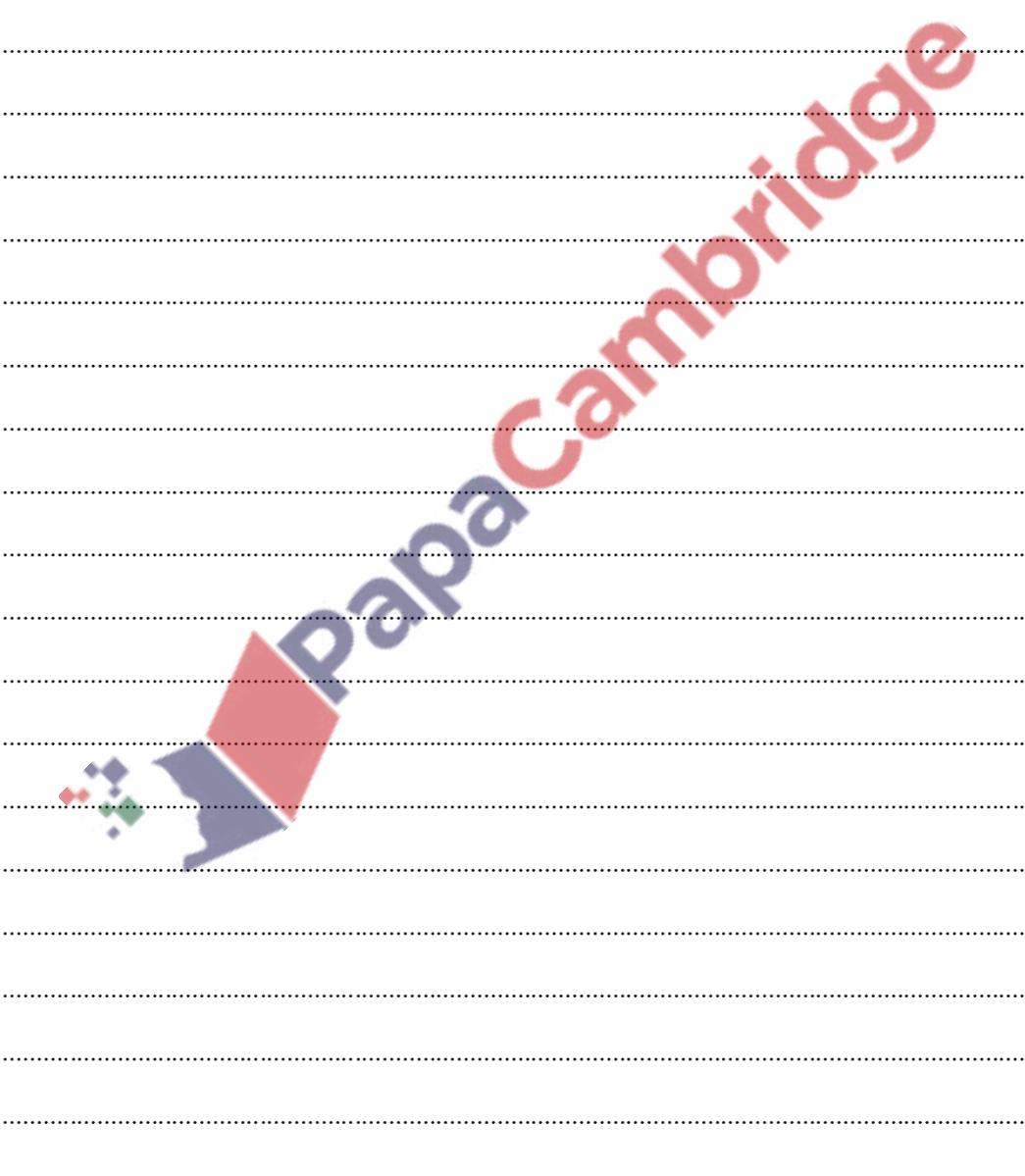
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- (c) State the equation of the curve which is the reflection of $y = f(x)$ in the x -axis. Give your answer in the form $y = a \cos 2x + b$, where a and b are constants. [1]

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193. 9709_s20_qp_11 Q: 7

(a) Prove the identity $\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = \frac{2}{\cos \theta}$. [3]



- (b) Hence solve the equation $\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = \frac{3}{\sin \theta}$, for $0 \leq \theta \leq 2\pi$. [3]

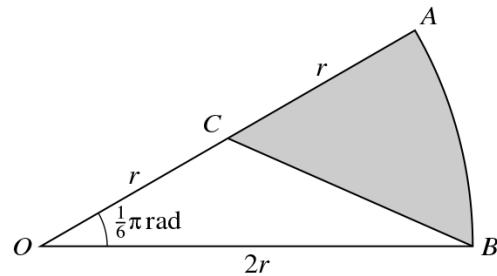
194. 9709_s20_qp_12_Q: 2

- (a) Express the equation $3 \cos \theta = 8 \tan \theta$ as a quadratic equation in $\sin \theta$. [3]

- (b) Hence find the acute angle, in degrees, for which $3 \cos \theta = 8 \tan \theta$. [2]

A large, semi-transparent watermark is positioned diagonally across the page. The word "Paper" is written in a bold, sans-serif font. The letters are partially cut off at the top right corner. The background of the watermark is a light gray gradient, and it features a subtle texture of small, colorful dots (red, green, blue) concentrated towards the bottom left.

195. 9709_s20_qp_12_Q: 7



In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA .

- (a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

- (b) Find the exact perimeter of the shaded region. [2]

- (c) Find the exact area of the shaded region. [3]

The logo for PapaCasa is displayed prominently at the top of the page. It features the brand name "PapaCasa" in a large, bold, sans-serif font. The letters are colored in a gradient: blue for "Papa" and red for "Casa". A small, colorful graphic of a house with a red roof and a blue base is positioned to the left of the text. Below the main title, there are several horizontal dotted lines, likely part of the page's header or design.

196. 9709_s20_qp_12 Q: 9

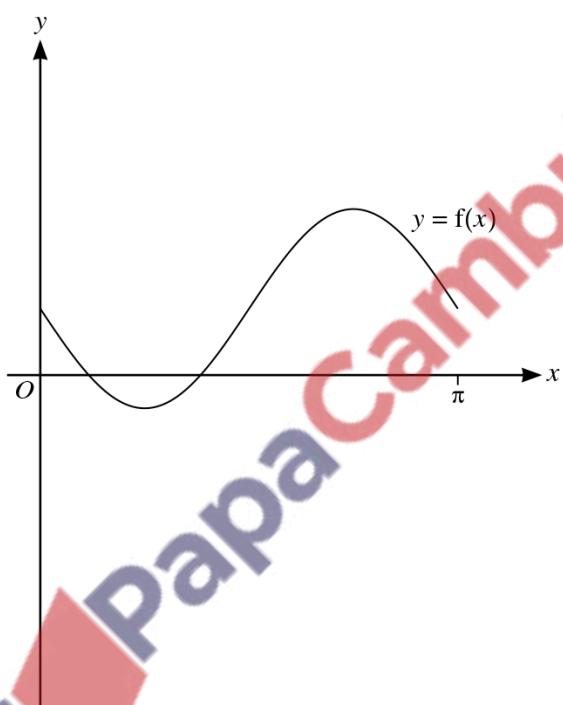
Functions f and g are such that

$$\begin{aligned}f(x) &= 2 - 3 \sin 2x \quad \text{for } 0 \leq x \leq \pi, \\g(x) &= -2f(x) \quad \text{for } 0 \leq x \leq \pi.\end{aligned}$$

- (a) State the ranges of f and g . [3]

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The diagram below shows the graph of $y = f(x)$.



- (b) Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

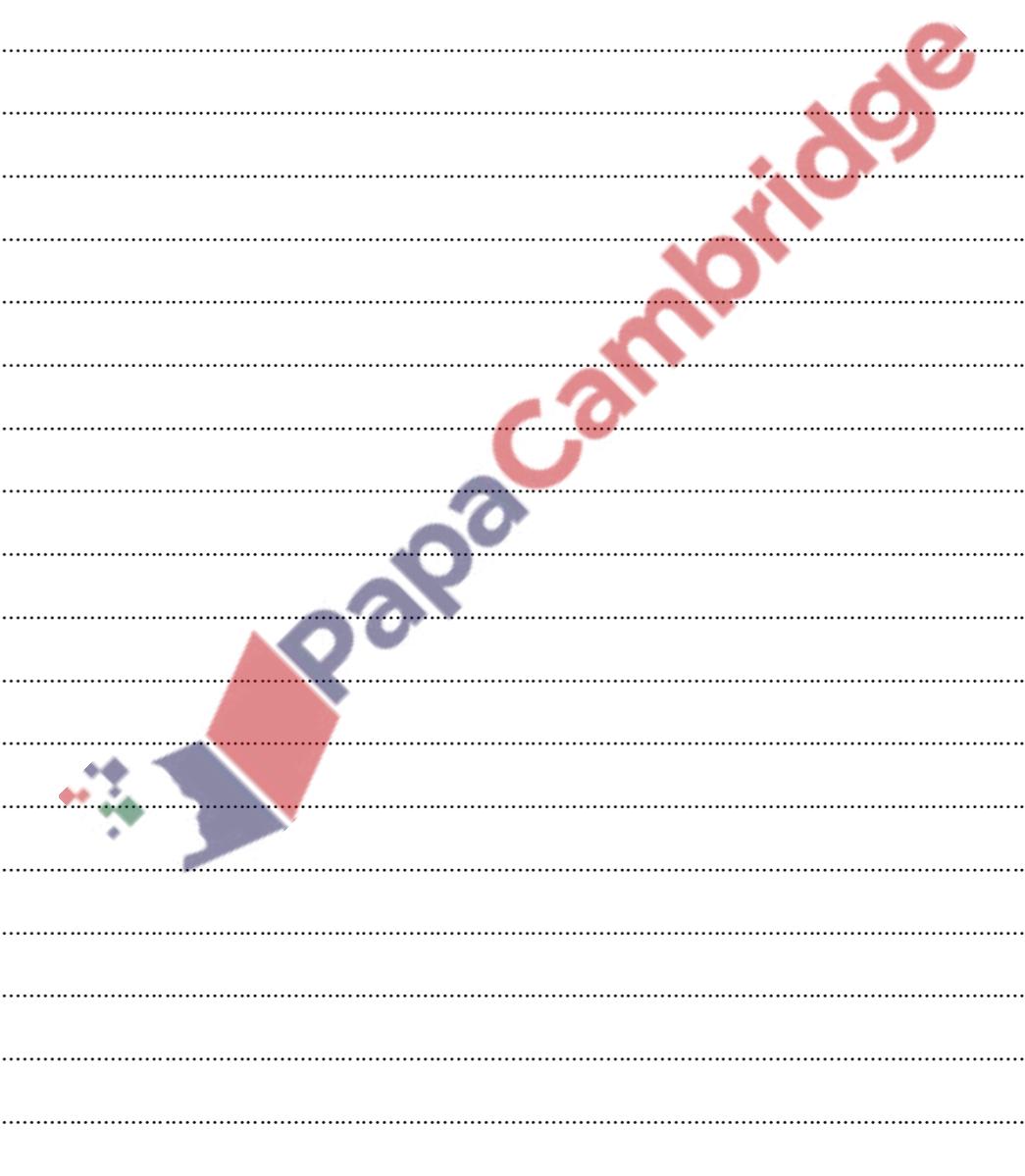
$$h(x) = g(x + \pi) \quad \text{for } -\pi \leq x \leq 0.$$

- (c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

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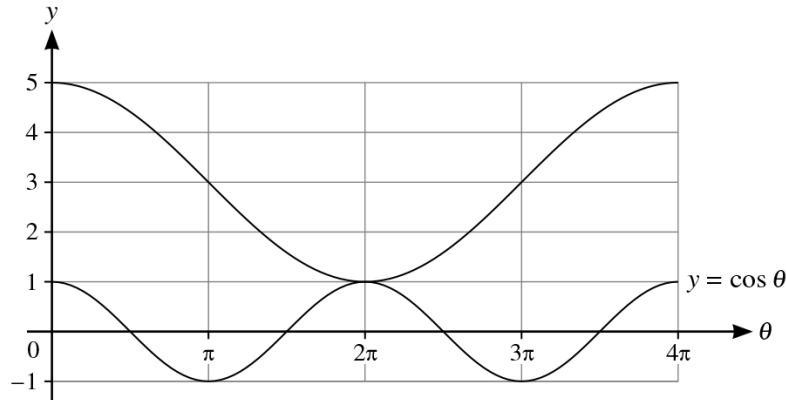
197. 9709_s20_qp_13 Q: 7

(a) Show that $\frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta} \equiv \frac{2}{\sin \theta \cos \theta}$. [4]



- (b) Hence solve the equation $\frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta} = \frac{6}{\tan \theta}$ for $0^\circ < \theta < 180^\circ$. [4]

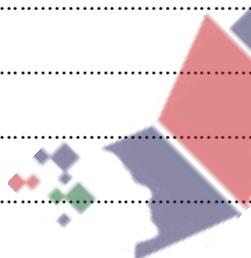
198. 9709_w20_qp_11 Q: 4



In the diagram, the lower curve has equation $y = \cos \theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos \theta$.

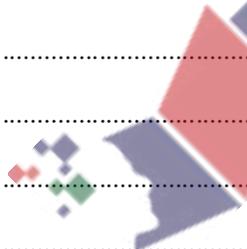
Find, in terms of a cosine function, the equation of the upper curve.

[3]



199. 9709_w20_qp_11 Q: 7

- (a) Show that $\frac{\sin \theta}{1 - \sin \theta} - \frac{\sin \theta}{1 + \sin \theta} \equiv 2 \tan^2 \theta$. [3]



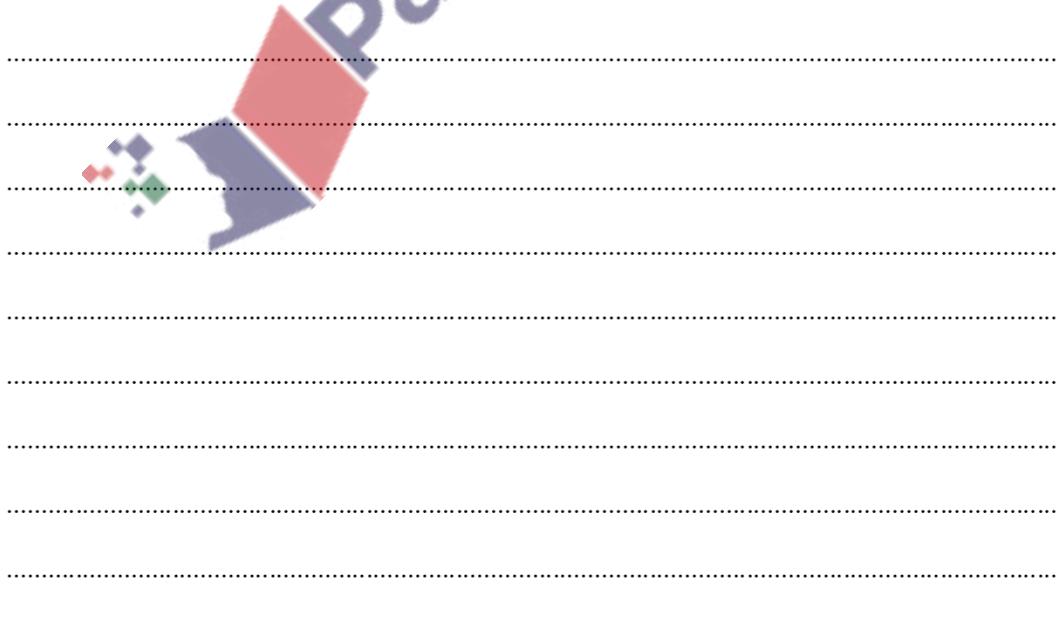
- (b) Hence solve the equation $\frac{\sin \theta}{1 - \sin \theta} - \frac{\sin \theta}{1 + \sin \theta} = 8$, for $0^\circ < \theta < 180^\circ$. [3]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, sans-serif font. The letters are colored in a gradient: "Papa" is in light blue, and "Cambridge" is in red. Below the text is a graphic element consisting of a red and blue abstract shape resembling a pen or brush tip, resting on a green and blue geometric base made of small squares.

200. 9709_w20_qp_12_Q: 6

- (a) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

- (b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2 \tan^2 x$ for $0^\circ \leq x \leq 180^\circ$. [2]



201. 9709_w20_qp_12 Q: 11

A curve has equation $y = 3 \cos 2x + 2$ for $0 \leq x \leq \pi$.

- (a) State the greatest and least values of
- y
- .

[2]

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- (b) Sketch the graph of
- $y = 3 \cos 2x + 2$
- for
- $0 \leq x \leq \pi$
- .

[2]

- (c) By considering the straight line
- $y = kx$
- , where
- k
- is a constant, state the number of solutions of the equation
- $3 \cos 2x + 2 = kx$
- for
- $0 \leq x \leq \pi$
- in each of the following cases.

- (i)
- $k = -3$

[1]

- (ii)
- $k = 1$

[1]

- (iii)
- $k = 3$

[1]

Functions f , g and h are defined for $x \in \mathbb{R}$ by

$$\begin{aligned}f(x) &= 3 \cos 2x + 2, \\g(x) &= f(2x) + 4, \\h(x) &= 2f\left(x + \frac{1}{2}\pi\right).\end{aligned}$$

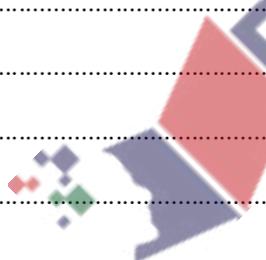
- (d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

- (e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

A large, semi-transparent watermark is positioned diagonally across the page. The word "Paper" is written in a bold, serif font, with each letter stacked on top of the next. The letters are a light grey color, allowing the background content to be visible through them.

202. 9709_w20_qp_13 Q: 3

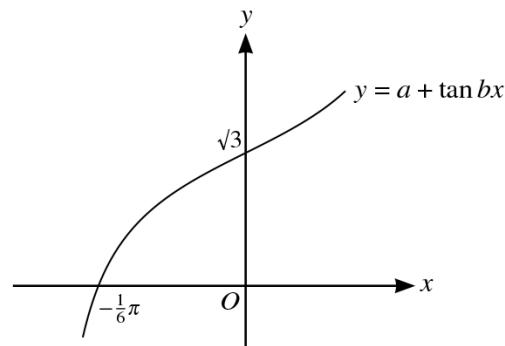
Solve the equation $3 \tan^2 \theta + 1 = \frac{2}{\tan^2 \theta}$ for $0^\circ < \theta < 180^\circ$. [5]



203. 9709_m19_qp_12 Q: 7

- (a) Solve the equation $3 \sin^2 2\theta + 8 \cos 2\theta = 0$ for $0^\circ \leq \theta \leq 180^\circ$. [5]

(b)

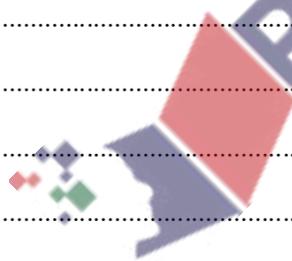


The diagram shows part of the graph of $y = a + \tan bx$, where x is measured in radians and a and b are constants. The curve intersects the x -axis at $(-\frac{1}{6}\pi, 0)$ and the y -axis at $(0, \sqrt{3})$. Find the values of a and b . [3]



204. 9709_s19_qp_11 Q: 6

- (i) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1 - \sin x}{1 + \sin x}$. [4]



- (ii) Hence solve the equation $\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leq x \leq \pi$. [3]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features a red icon of a building with a flag-like element at the top left. To the right of the icon, the words "PapaCambridge" are written in a red, sans-serif font, with each word on a new line and slightly overlapping the other.

205. 9709_s19_qp_11 Q: 9

The function f is defined by $f(x) = 2 - 3 \cos x$ for $0 \leq x \leq 2\pi$.

- (i) State the range of f . [2]

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- (ii) Sketch the graph of $y = f(x)$. [2]



The function g is defined by $g(x) = 2 - 3 \cos x$ for $0 \leq x \leq p$, where p is a constant.

- (iii) State the largest value of p for which g has an inverse. [1]

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- (iv) For this value of p , find an expression for $g^{-1}(x)$. [2]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, sans-serif font. The letters are primarily red, with some blue used for the 'P' and 'C'. Below the text is a graphic element consisting of a red and blue abstract shape that resembles a pen or marker tip, with a trail of small dots extending downwards and to the left.

206. 9709 - s19 - qp - 12 Q: 4

Angle x is such that $\sin x = a + b$ and $\cos x = a - b$, where a and b are constants.

- (i) Show that $a^2 + b^2$ has a constant value for all values of x . [3]

- (ii) In the case where $\tan x = 2$, express a in terms of b . [2]

207. 9709_s19_qp_12 Q: 6

The equation of a curve is $y = 3 \cos 2x$ and the equation of a line is $2y + \frac{3x}{\pi} = 5$.

- (i) State the smallest and largest values of y for both the curve and the line for $0 \leq x \leq 2\pi$. [3]

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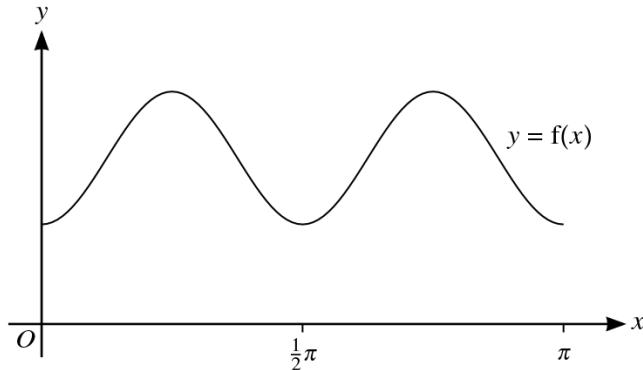
- (ii) Sketch, on the same diagram, the graphs of $y = 3 \cos 2x$ and $2y + \frac{3x}{\pi} = 5$ for $0 \leq x \leq 2\pi$. [3]



- (iii) State the number of solutions of the equation $6 \cos 2x = 5 - \frac{3x}{\pi}$ for $0 \leq x \leq 2\pi$. [1]

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208. 9709_s19_qp_13 Q: 9



The function $f : x \mapsto p \sin^2 2x + q$ is defined for $0 \leq x \leq \pi$, where p and q are positive constants. The diagram shows the graph of $y = f(x)$.

- (i) In terms of p and q , state the range of f . [2]

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- (ii) State the number of solutions of the following equations.

(a) $f(x) = p + q$ [1]

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(b) $f(x) = q$ [1]

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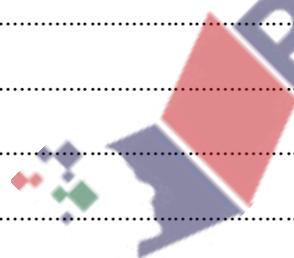
(c) $f(x) = \frac{1}{2}p + q$ [1]

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- (iii) For the case where $p = 3$ and $q = 2$, solve the equation $f(x) = 4$, showing all necessary working. [5]

209. 9709_w19_qp_11 Q: 5

- (i) Given that $4 \tan x + 3 \cos x + \frac{1}{\cos x} = 0$, show, without using a calculator, that $\sin x = -\frac{2}{3}$. [3]



(ii) Hence, showing all necessary working, solve the equation

$$4 \tan(2x - 20^\circ) + 3 \cos(2x - 20^\circ) + \frac{1}{\cos(2x - 20^\circ)} = 0$$

for $0^\circ \leq x \leq 180^\circ$.

[4]



210. 9709_w19_qp_12 Q: 6

- (a) Given that $x > 0$, find the two smallest values of x , in radians, for which $3 \tan(2x + 1) = 1$. Show all necessary working. [4]

- (b) The function $f : x \mapsto 3\cos^2x - 2\sin^2x$ is defined for $0 \leq x \leq \pi$.

(i) Express $f(x)$ in the form $a\cos^2x + b$, where a and b are constants. [1]

- (ii) Find the range of f . [2]

211. 9709_w19_qp_13 Q: 7

- (i) Show that the equation $3\cos^4 \theta + 4\sin^2 \theta - 3 = 0$ can be expressed as $3x^2 - 4x + 1 = 0$, where $x = \cos^2 \theta$. [2]

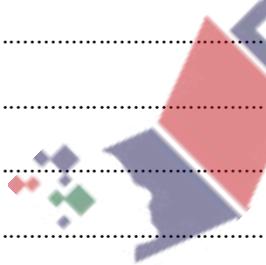
A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the word "Papacambridge" in a bold, sans-serif font, with "Papac" in blue and "Cambridge" in red. A stylized graphic of a pen tip is integrated into the letter "P". Below the main text, there is smaller, illegible text that appears to be a copyright notice.

- (ii) Hence solve the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ for $0^\circ \leq \theta \leq 180^\circ$. [5]

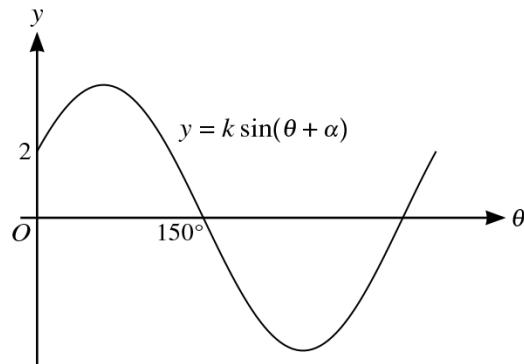
A large, diagonal watermark logo is positioned across the page. The logo features a stylized rocket ship with a red nose cone and a blue body, angled upwards from the bottom left. At the base of the rocket is a circular emblem containing a green and blue geometric pattern. The entire logo is rendered in a light gray color.

212. 9709_m18_qp_12 Q: 5

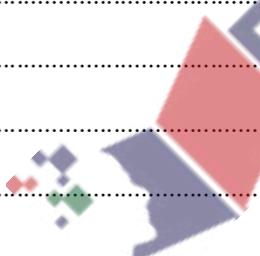
- (a) Express the equation $\frac{5 + 2 \tan x}{3 + 2 \tan x} = 1 + \tan x$ as a quadratic equation in $\tan x$ and hence solve the equation for $0 \leq x \leq \pi$. [4]



(b)

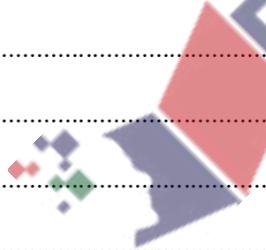


The diagram shows part of the graph of $y = k \sin(\theta + \alpha)$, where k and α are constants and $0^\circ < \alpha < 180^\circ$. Find the value of α and the value of k . [2]



213. 9709_s18_qp_11 Q: 4

- (i) Prove the identity $(\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta) \equiv \sin^3 \theta + \cos^3 \theta$. [3]



- (ii) Hence solve the equation $(\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta) = 3 \cos^3 \theta$ for $0^\circ \leq \theta \leq 360^\circ$. [3]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, sans-serif font. The letters are primarily red, with some blue used for the 'P' and 'C'. Below the text is a graphic element consisting of a red and blue abstract shape that resembles a pen or pencil, with small green and red dots scattered around its base.

214. 9709_s18_qp_12_Q: 4

The function f is such that $f(x) = a + b \cos x$ for $0 \leq x \leq 2\pi$. It is given that $f\left(\frac{1}{3}\pi\right) = 5$ and $f(\pi) = 11$.

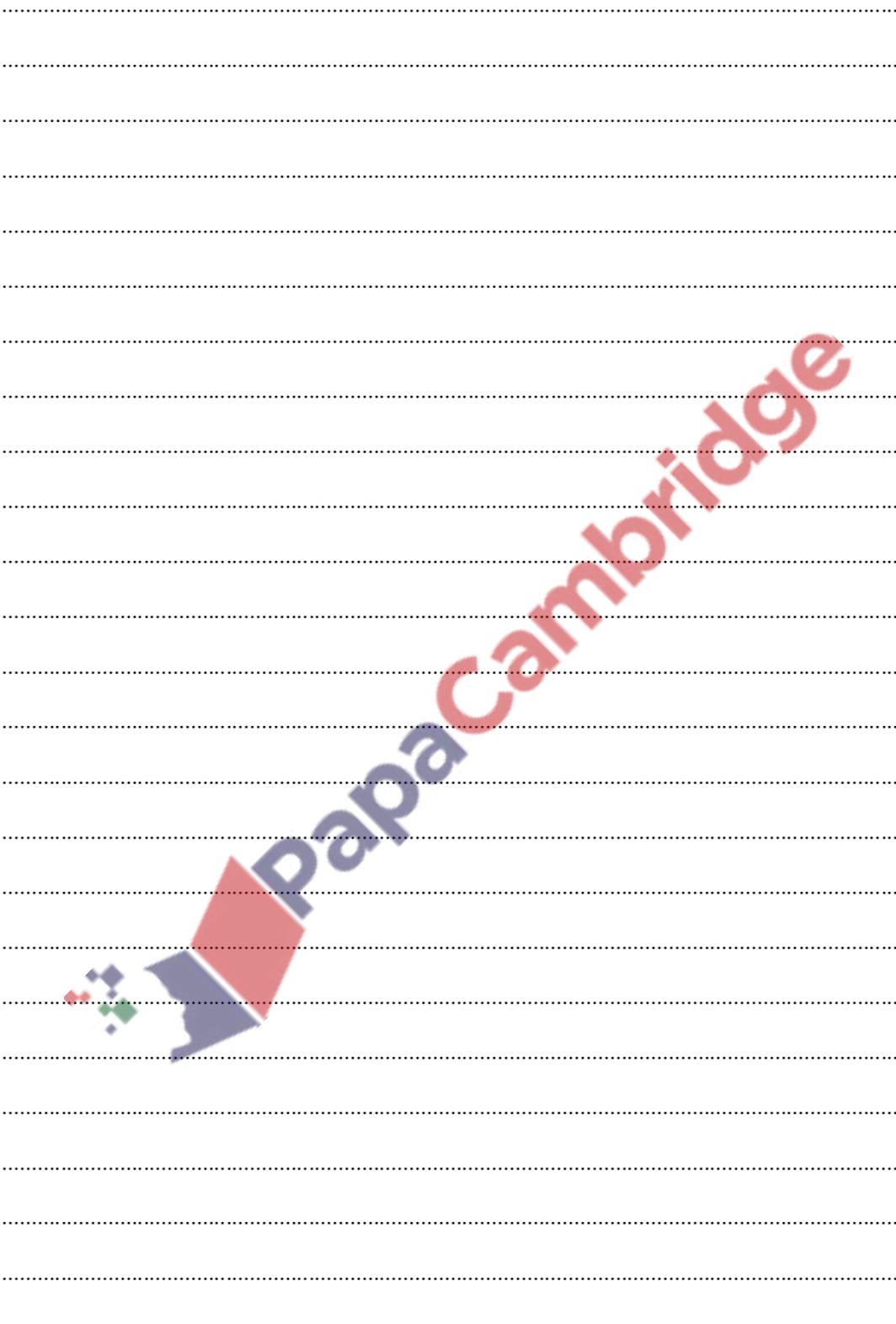
- (i) Find the values of the constants a and b . [3]

- (ii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [3]

The image features a large, stylized graphic in the upper left corner. The graphic consists of a red trapezoid pointing upwards, a blue trapezoid pointing downwards, and a smaller blue triangle pointing right. To the right of this graphic, the word "Papa" is written in a bold, slanted, grey sans-serif font. The background of the page is white with horizontal grey dotted lines.

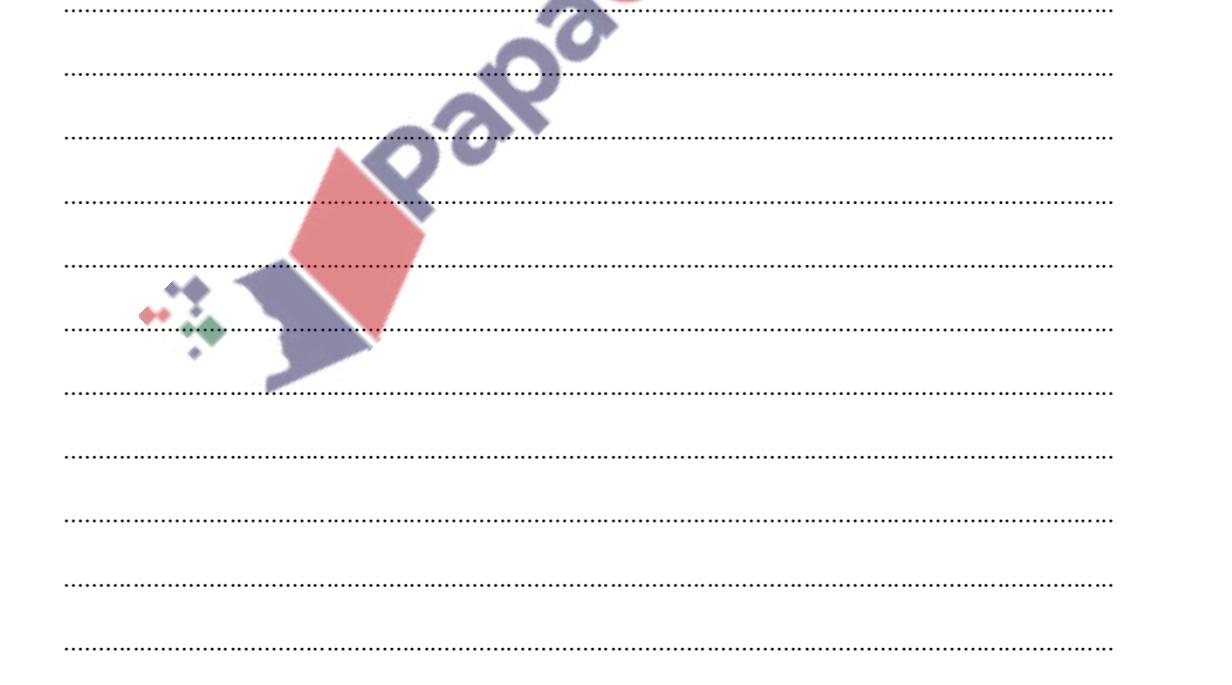
215. 9709_s18_qp_12 Q: 10

- (i) Solve the equation $2 \cos x + 3 \sin x = 0$, for $0^\circ \leq x \leq 360^\circ$. [3]



- (ii) Sketch, on the same diagram, the graphs of $y = 2 \cos x$ and $y = -3 \sin x$ for $0^\circ \leq x \leq 360^\circ$. [3]

(iii) Use your answers to parts (i) and (ii) to find the set of values of x for $0^\circ \leq x \leq 360^\circ$ for which $2 \cos x + 3 \sin x > 0$. [2]



216. 9709_s18_qp_13 Q: 7

- (a) (i) Express $\frac{\tan^2 \theta - 1}{\tan^2 \theta + 1}$ in the form $a \sin^2 \theta + b$, where a and b are constants to be found. [3]

- (ii) Hence, or otherwise, and showing all necessary working, solve the equation

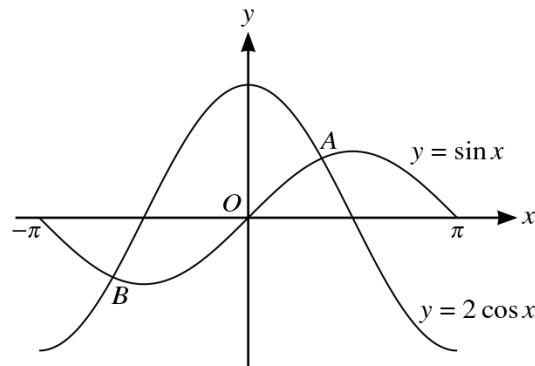
$$\frac{\tan^2 \theta - 1}{\tan^2 \theta + 1} = \frac{1}{4}$$

for $-90^\circ \leq \theta \leq 0^\circ$.

[2]



(b)



The diagram shows the graphs of $y = \sin x$ and $y = 2 \cos x$ for $-\pi \leq x \leq \pi$. The graphs intersect at the points A and B .

- (i) Find the x -coordinate of A .

[2]

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- (ii) Find the y -coordinate of B .

[2]

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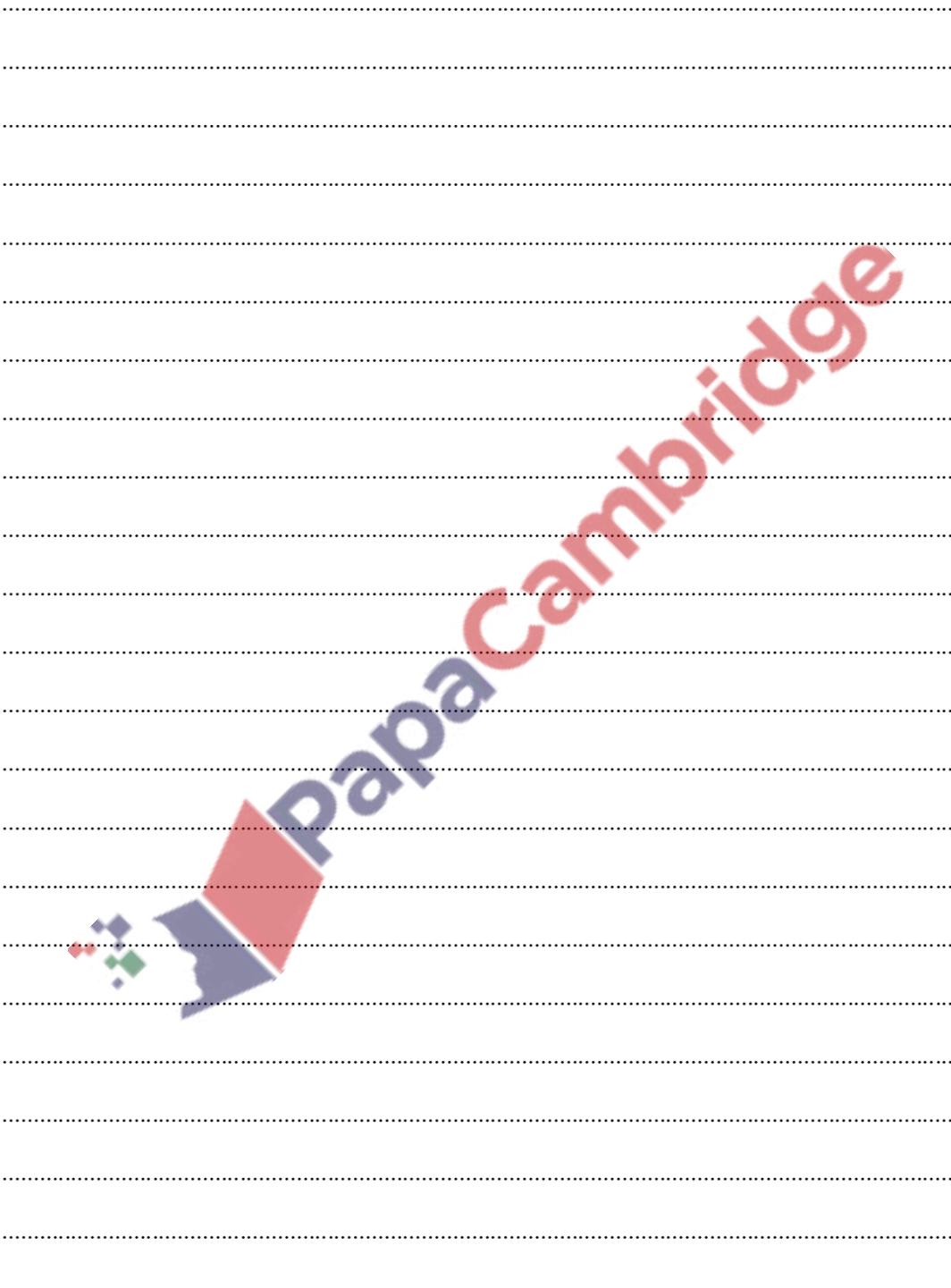
217. 9709_w18_qp_11 Q: 5

- (i) Show that the equation

$$\frac{\cos \theta - 4}{\sin \theta} - \frac{4 \sin \theta}{5 \cos \theta - 2} = 0$$

may be expressed as $9 \cos^2 \theta - 22 \cos \theta + 4 = 0$.

[3]



(ii) Hence solve the equation

$$\frac{\cos \theta - 4}{\sin \theta} - \frac{4 \sin \theta}{5 \cos \theta - 2} = 0$$

for $0^\circ \leq \theta \leq 360^\circ$.

[3]



218. 9709_w18_qp_12 Q: 4

Functions f and g are defined by

$$f : x \mapsto 2 - 3 \cos x \quad \text{for } 0 \leq x \leq 2\pi,$$

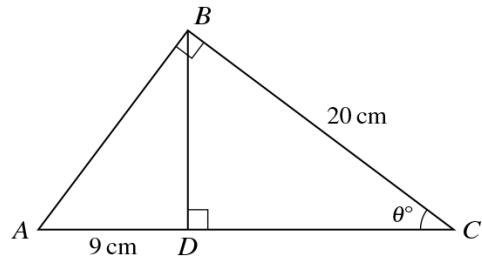
$$g : x \mapsto \frac{1}{2}x \quad \text{for } 0 \leq x \leq 2\pi.$$

- (i) Solve the equation $fg(x) = 1$. [3]

- (ii) Sketch the graph of $y = f(x)$. [3]

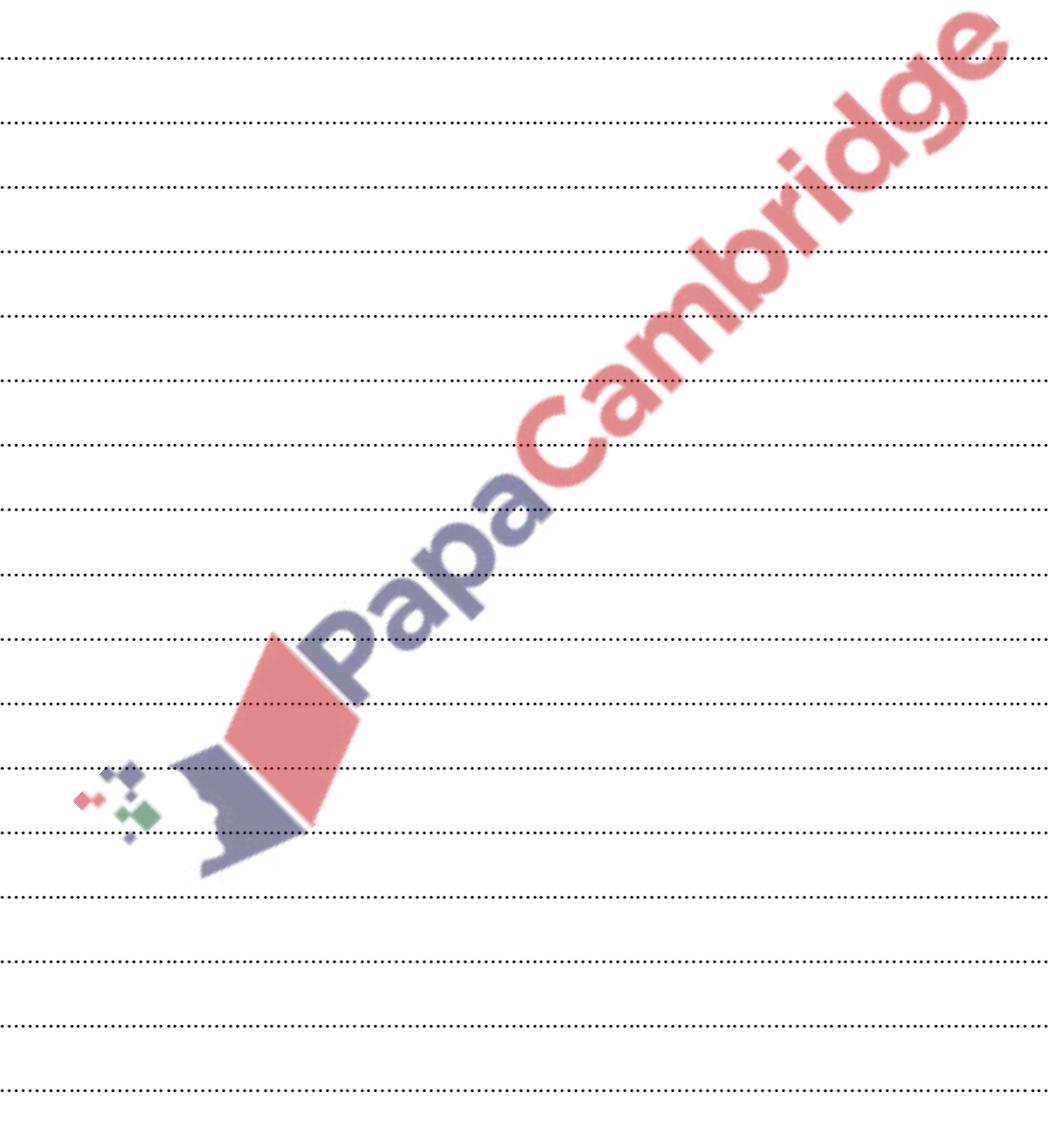


219. 9709_w18_qp_12 Q: 6



The diagram shows a triangle ABC in which $BC = 20\text{ cm}$ and angle $ABC = 90^\circ$. The perpendicular from B to AC meets AC at D and $AD = 9\text{ cm}$. Angle $BCA = \theta^\circ$.

- (i) By expressing the length of BD in terms of θ in each of the triangles ABD and DBC , show that $20 \sin^2 \theta = 9 \cos \theta$. [4]

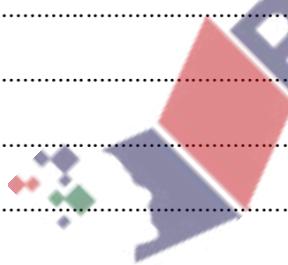


- (ii) Hence, showing all necessary working, calculate θ . [3]

A large, diagonal watermark logo is positioned across the page. The logo consists of a stylized, three-dimensional geometric shape composed of red, blue, and white facets. To the left of this shape, there are small, scattered colored dots in shades of red, green, and blue. The text "PapaCambridge" is written in a bold, sans-serif font, with "Papa" in grey and "Cambridge" in red, running diagonally from the bottom-left towards the top-right.

220. 9709_w18_qp_13 Q: 7

(i) Show that $\frac{\tan \theta + 1}{1 + \cos \theta} + \frac{\tan \theta - 1}{1 - \cos \theta} \equiv \frac{2(\tan \theta - \cos \theta)}{\sin^2 \theta}$. [3]

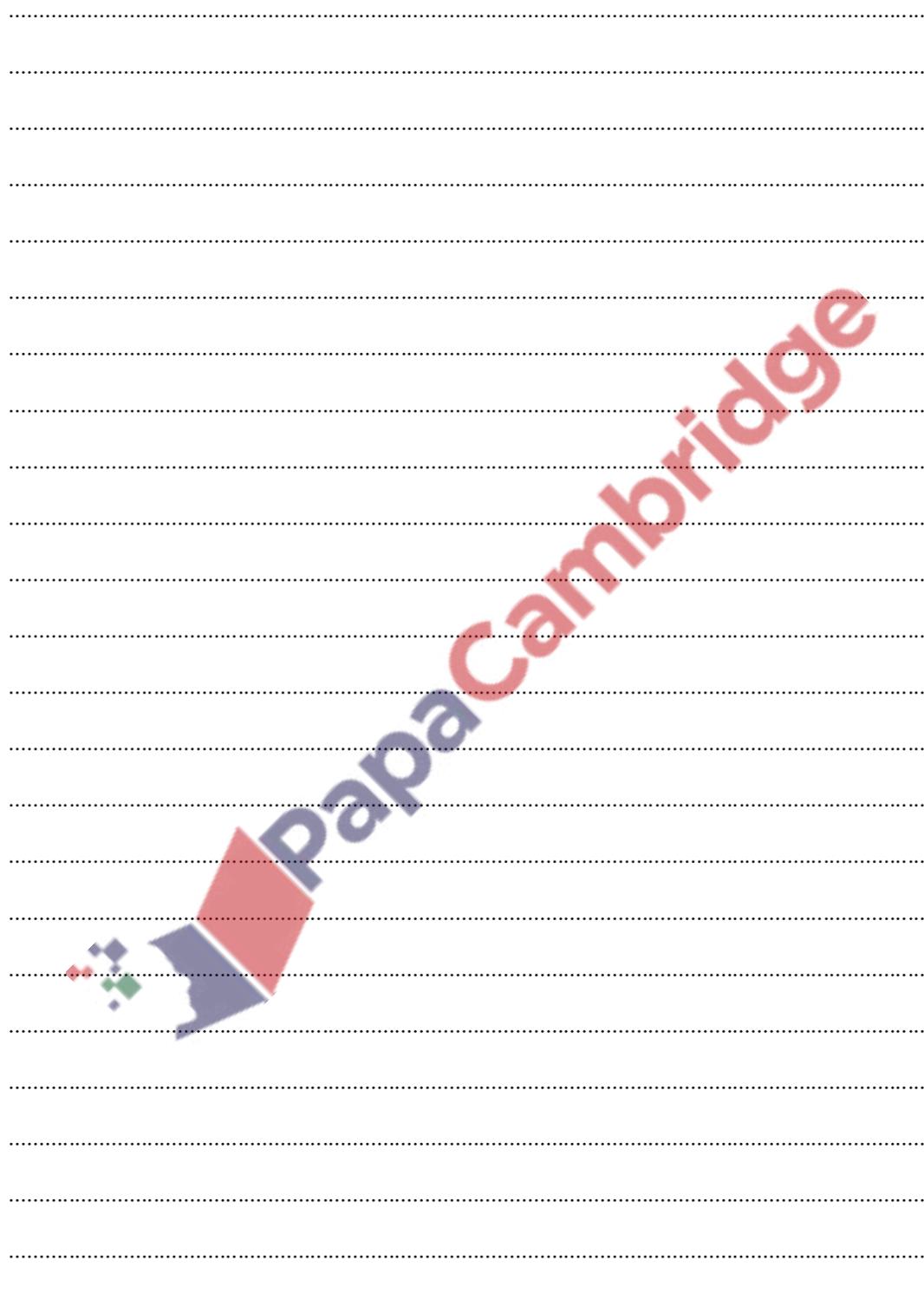


(ii) Hence, showing all necessary working, solve the equation

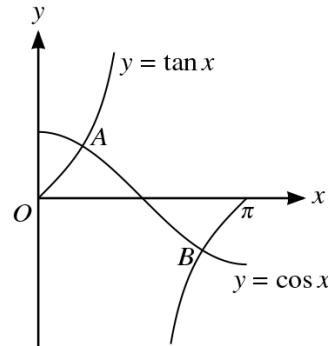
$$\frac{\tan \theta + 1}{1 + \cos \theta} + \frac{\tan \theta - 1}{1 - \cos \theta} = 0$$

for $0^\circ < \theta < 90^\circ$.

[4]



221. 9709_m17_qp_12 Q: 5



The diagram shows the graphs of $y = \tan x$ and $y = \cos x$ for $0 \leq x \leq \pi$. The graphs intersect at points A and B.

- (i) Find by calculation the x -coordinate of A . [4]



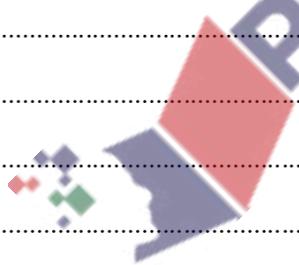
338

- (ii) Find by calculation the coordinates of B . [3]

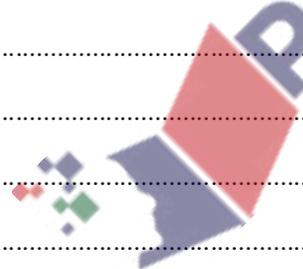


222. 9709_s17_qp_11 Q: 3

- (i) Prove the identity $\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} = \frac{2}{\sin \theta}$. [3]



(ii) Hence solve the equation $\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} = \frac{3}{\cos \theta}$ for $0^\circ \leq \theta \leq 360^\circ$. [3]



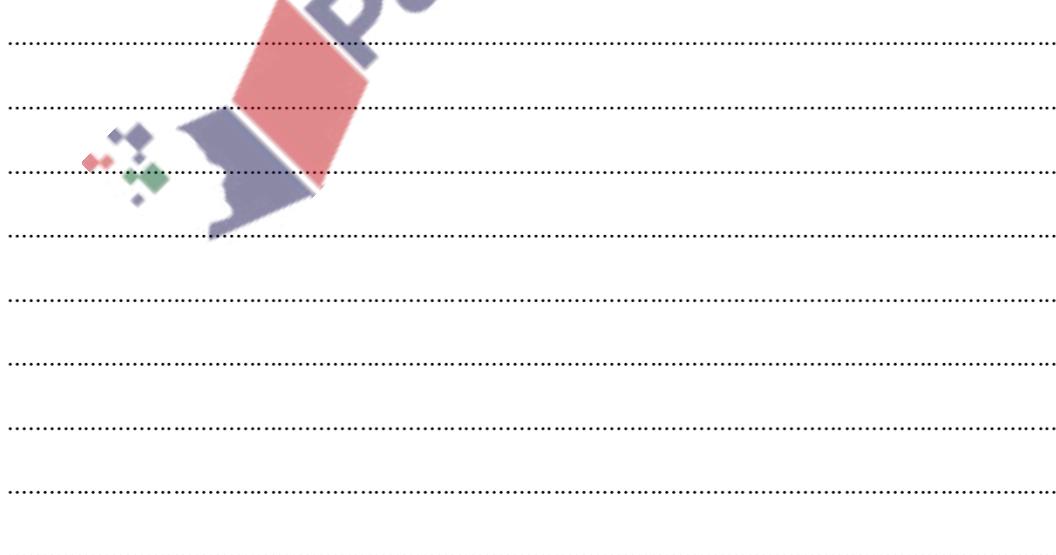
223. 9709_s17_qp_11 Q: 5

The equation of a curve is $y = 2 \cos x$.

- (i) Sketch the graph of $y = 2 \cos x$ for $-\pi \leq x \leq \pi$, stating the coordinates of the point of intersection with the y -axis. [2]

Points P and Q lie on the curve and have x -coordinates of $\frac{1}{3}\pi$ and π respectively.

- (ii) Find the length of PQ correct to 1 decimal place. [2]



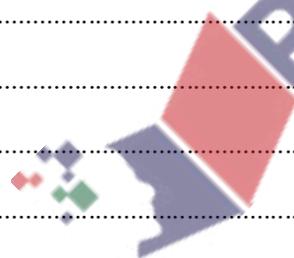
The line through P and Q meets the x -axis at $H(h, 0)$ and the y -axis at $K(0, k)$.

- (iii) Show that $h = \frac{5}{9}\pi$ and find the value of k . [3]

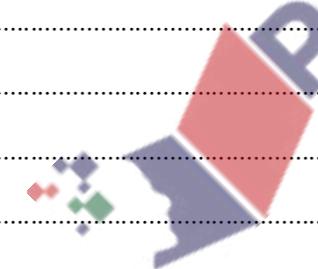


224. 9709_s17_qp_12_Q: 3

(i) Prove the identity $\left(\frac{1}{\cos \theta} - \tan \theta\right)^2 \equiv \frac{1 - \sin \theta}{1 + \sin \theta}$. [3]



(ii) Hence solve the equation $\left(\frac{1}{\cos \theta} - \tan \theta\right)^2 = \frac{1}{2}$, for $0^\circ \leq \theta \leq 360^\circ$. [3]



225. 9709 - s17 - qp - 12 Q: 10

The function f is defined by $f(x) = 3 \tan\left(\frac{1}{2}x\right) - 2$, for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

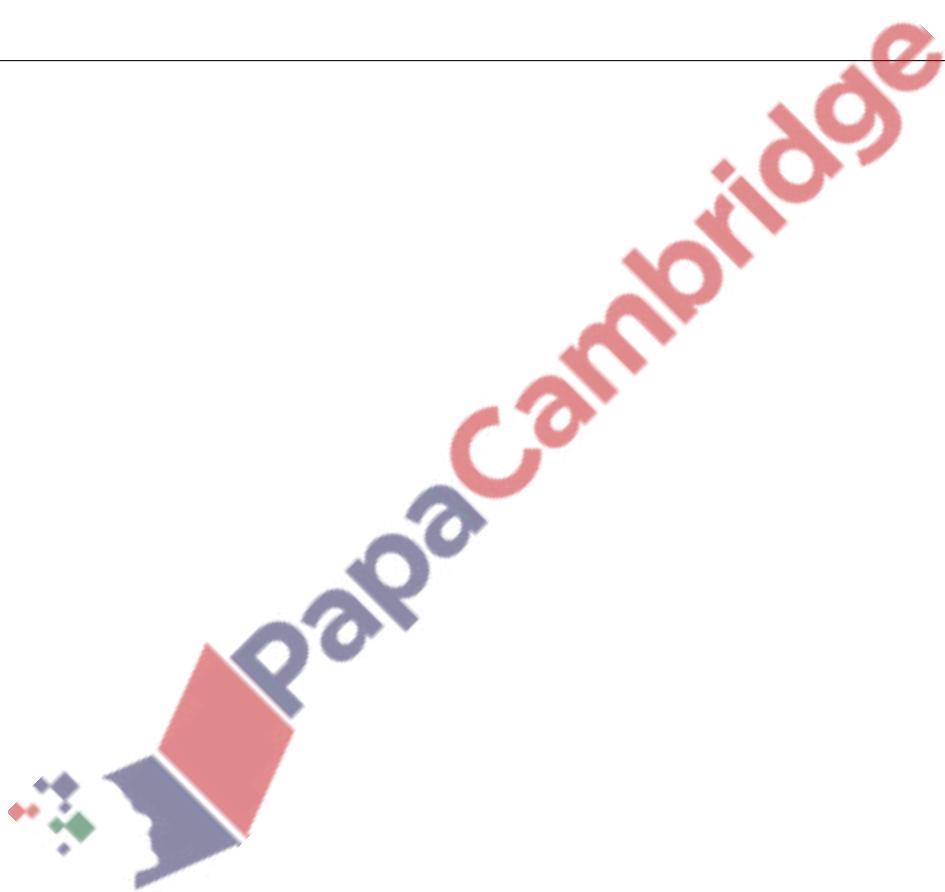
- (i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]

- (ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1} . [5]

The logo for PapaCar features the word "PapaCar" in a large, stylized font where the letters are slanted diagonally upwards from left to right. The letters are colored in a gradient: red for 'P', blue for 'a', grey for 'P', and red again for 'a' and 'r'. Below the main text is a graphic element consisting of a blue downward-pointing triangle pointing towards a red square. To the left of this graphic is a small cluster of geometric shapes in red, green, and blue.

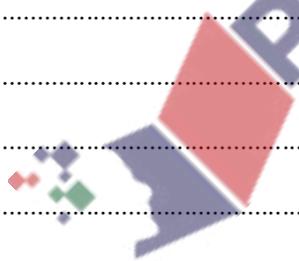
(iii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

[3]



226. 9709_s17_qp_13 Q: 5

- (i) Show that the equation $\frac{2 \sin \theta + \cos \theta}{\sin \theta + \cos \theta} = 2 \tan \theta$ may be expressed as $\cos^2 \theta = 2 \sin^2 \theta$. [3]

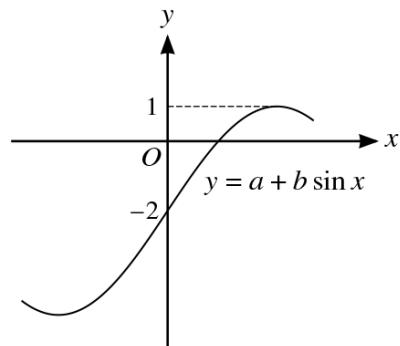


- (ii) Hence solve the equation $\frac{2 \sin \theta + \cos \theta}{\sin \theta + \cos \theta} = 2 \tan \theta$ for $0^\circ < \theta < 180^\circ$. [3]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the word "PapaCambridge" in a stylized, rounded font. The letters are primarily red, with some blue and green accents. Below the text is a graphic element consisting of a red diamond shape pointing upwards, with a blue base and small green and red dots scattered around it.



227. 9709_w17_qp_11 Q: 7



The diagram shows part of the graph of $y = a + b \sin x$. Find the values of the constants a and b . [2]



- (b) (i)** Show that the equation

$$(\sin \theta + 2 \cos \theta)(1 + \sin \theta - \cos \theta) = \sin \theta(1 + \cos \theta)$$

may be expressed as $3\cos^2\theta - 2\cos\theta - 1 = 0$.

[3]

- (ii) Hence solve the equation

$$(\sin \theta + 2 \cos \theta)(1 + \sin \theta - \cos \theta) = \sin \theta(1 + \cos \theta)$$

for $-180^\circ \leq \theta \leq 180^\circ$.

[4]

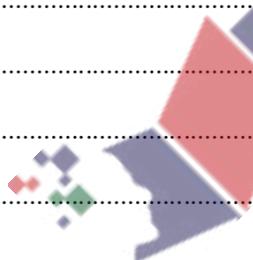
A red and blue eraser with a small cluster of colorful geometric shapes.

228. 9709_w17_qp_12 Q: 5

- (i) Show that the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ can be expressed as

$$2\cos^2 2x + 3\cos 2x + 1 = 0.$$

[3]



- (ii) Hence solve the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ for $0^\circ \leq x \leq 180^\circ$. [4]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the word "PapaCambridge" in a bold, sans-serif font. The letters are colored in a gradient from red at the top to blue at the bottom. Below the text is a stylized graphic of a pen or marker. The barrel of the pen is red with a white band near the cap. The cap is blue with a white band. A small green and red geometric pattern is located at the base of the pen's barrel.

229. 9709_w17_qp_12 Q: 6

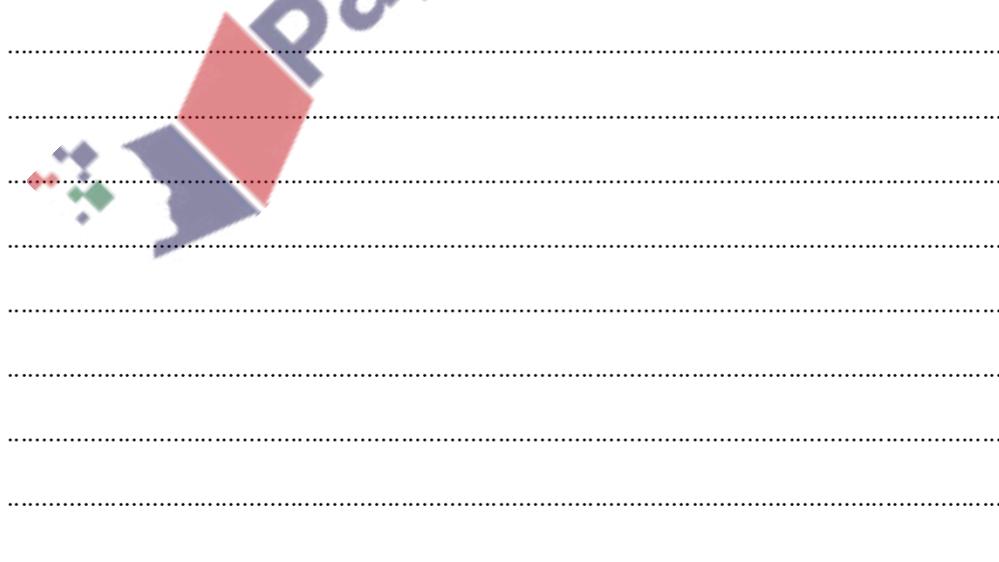
- (a) The function f , defined by $f : x \mapsto a + b \sin x$ for $x \in \mathbb{R}$, is such that $f\left(\frac{1}{6}\pi\right) = 4$ and $f\left(\frac{1}{2}\pi\right) = 3$.

- (i) Find the values of the constants a and b .

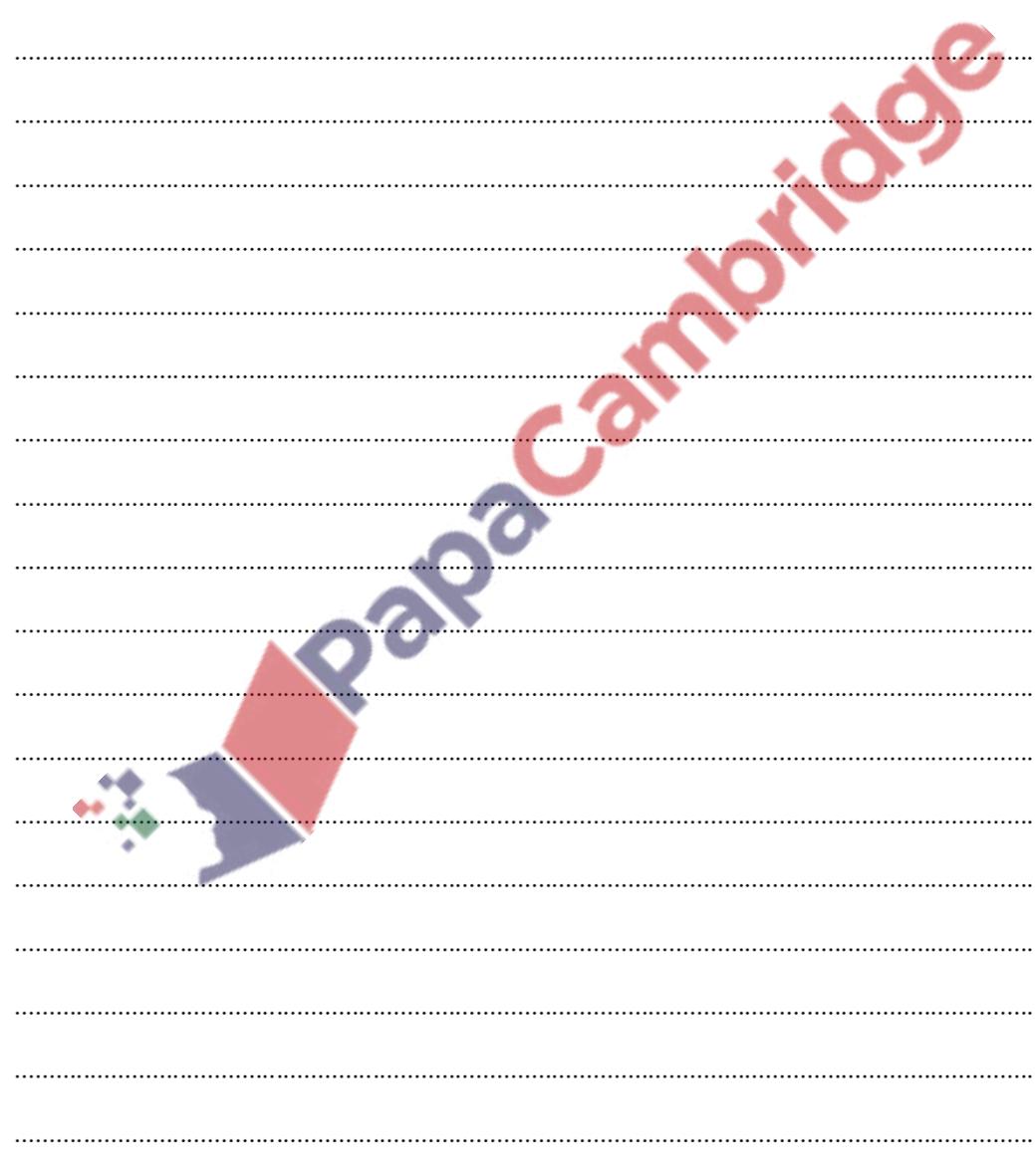
[3]

- (ii) Evaluate $ff(0)$.

[2]

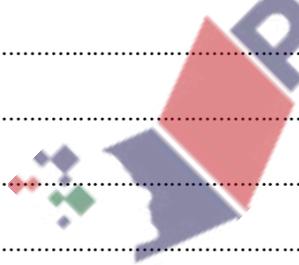


- (b) The function g is defined by $g : x \mapsto c + d \sin x$ for $x \in \mathbb{R}$. The range of g is given by $-4 \leq g(x) \leq 10$. Find the values of the constants c and d . [3]

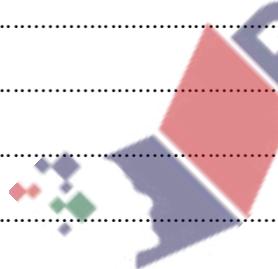


230. 9709_w17_qp_13 Q: 5

- (i) Show that the equation $\frac{\cos \theta + 4}{\sin \theta + 1} + 5 \sin \theta - 5 = 0$ may be expressed as $5 \cos^2 \theta - \cos \theta - 4 = 0$. [3]



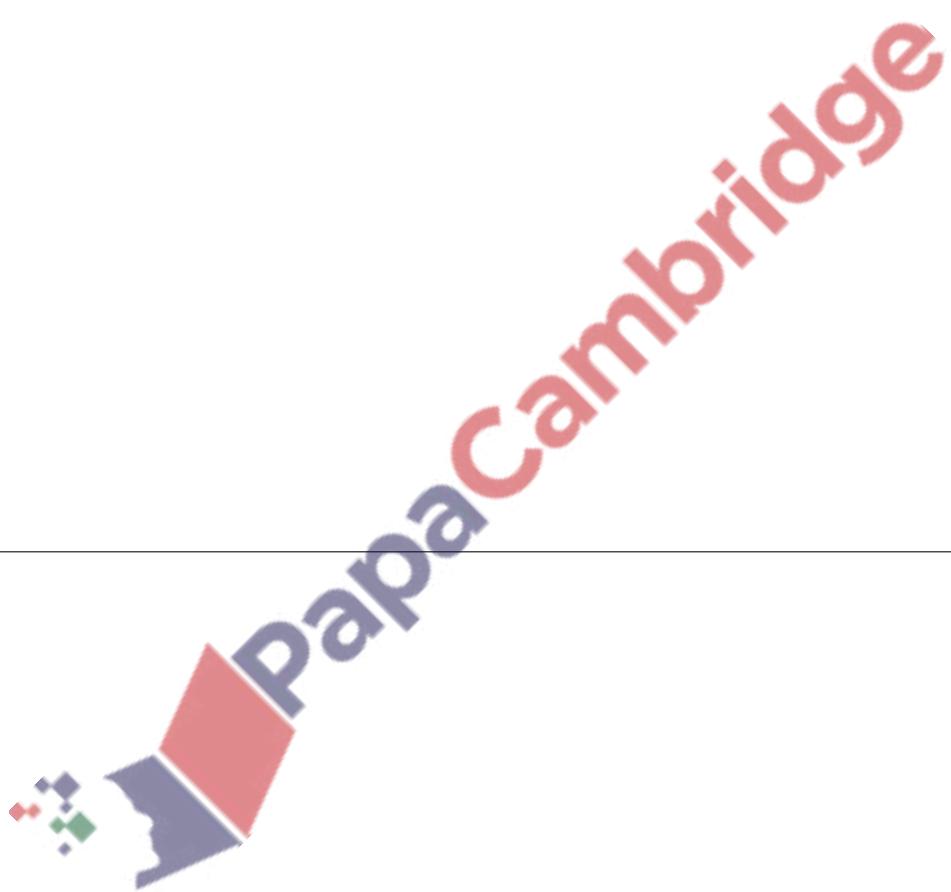
- (ii) Hence solve the equation $\frac{\cos \theta + 4}{\sin \theta + 1} + 5 \sin \theta - 5 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. [4]



231. 9709 _ m16 _ qp _ 12 Q: 4

(a) Solve the equation $\sin^{-1}(3x) = -\frac{1}{3}\pi$, giving the solution in an exact form. [2]

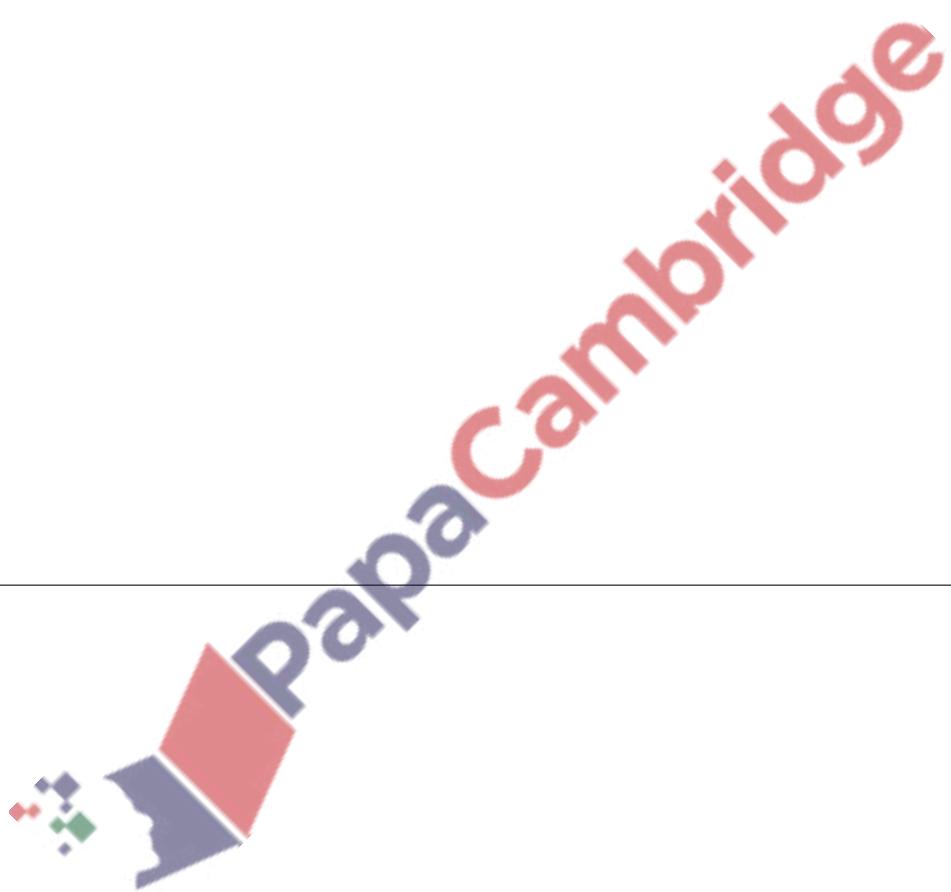
(b) Solve, by factorising, the equation $2 \cos \theta \sin \theta - 2 \cos \theta - \sin \theta + 1 = 0$ for $0 \leq \theta \leq \pi$. [4]



232. 9709_s16_qp_11 Q: 2

Solve the equation $3 \sin^2 \theta = 4 \cos \theta - 1$ for $0^\circ \leq \theta \leq 360^\circ$.

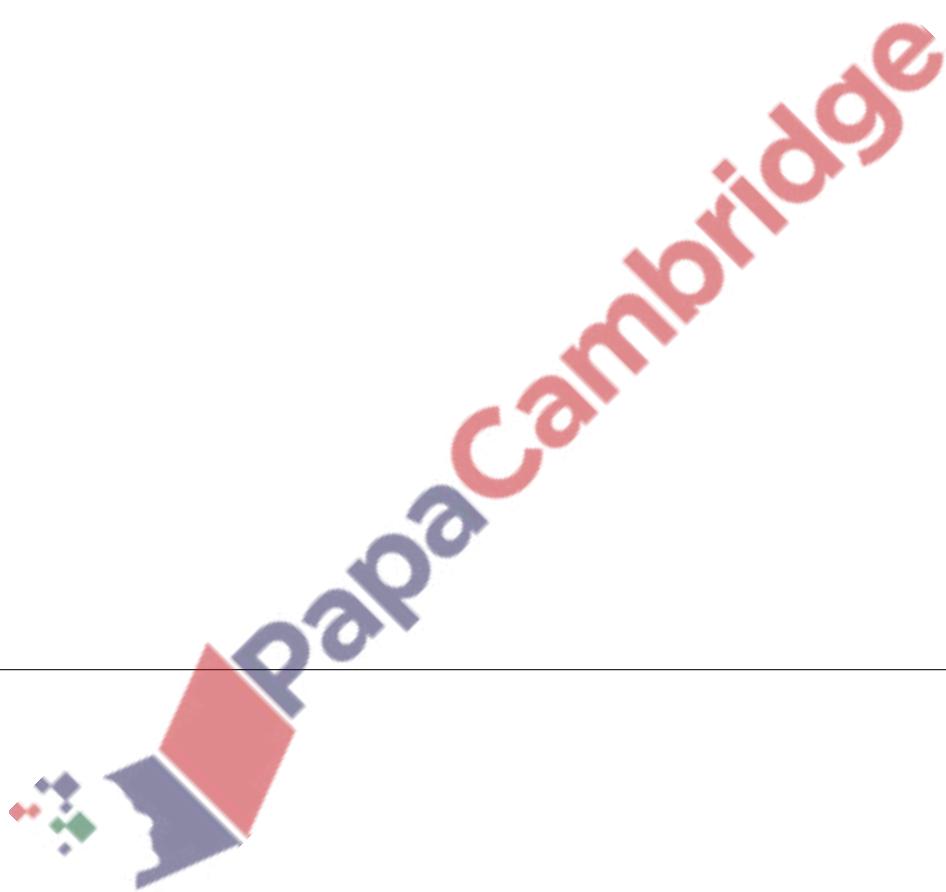
[4]



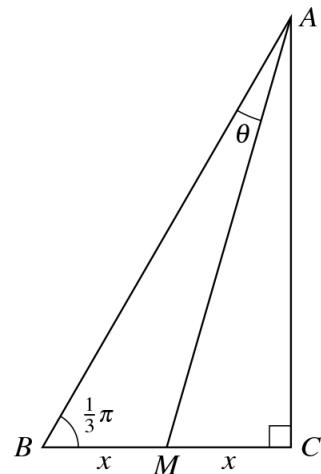
233. 9709_s16_qp_11 Q: 11

The function f is defined by $f : x \mapsto 4 \sin x - 1$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

- (i) State the range of f . [2]
- (ii) Find the coordinates of the points at which the curve $y = f(x)$ intersects the coordinate axes. [3]
- (iii) Sketch the graph of $y = f(x)$. [2]
- (iv) Obtain an expression for $f^{-1}(x)$, stating both the domain and range of f^{-1} . [4]



234. 9709_s16_qp_12 Q: 5



In the diagram, triangle ABC is right-angled at C and M is the mid-point of BC . It is given that angle $ABC = \frac{1}{3}\pi$ radians and angle $BAM = \theta$ radians. Denoting the lengths of BM and MC by x ,

- (i) find AM in terms of x , [3]
- (ii) show that $\theta = \frac{1}{6}\pi - \tan^{-1} \left(\frac{1}{2\sqrt{3}} \right)$. [2]

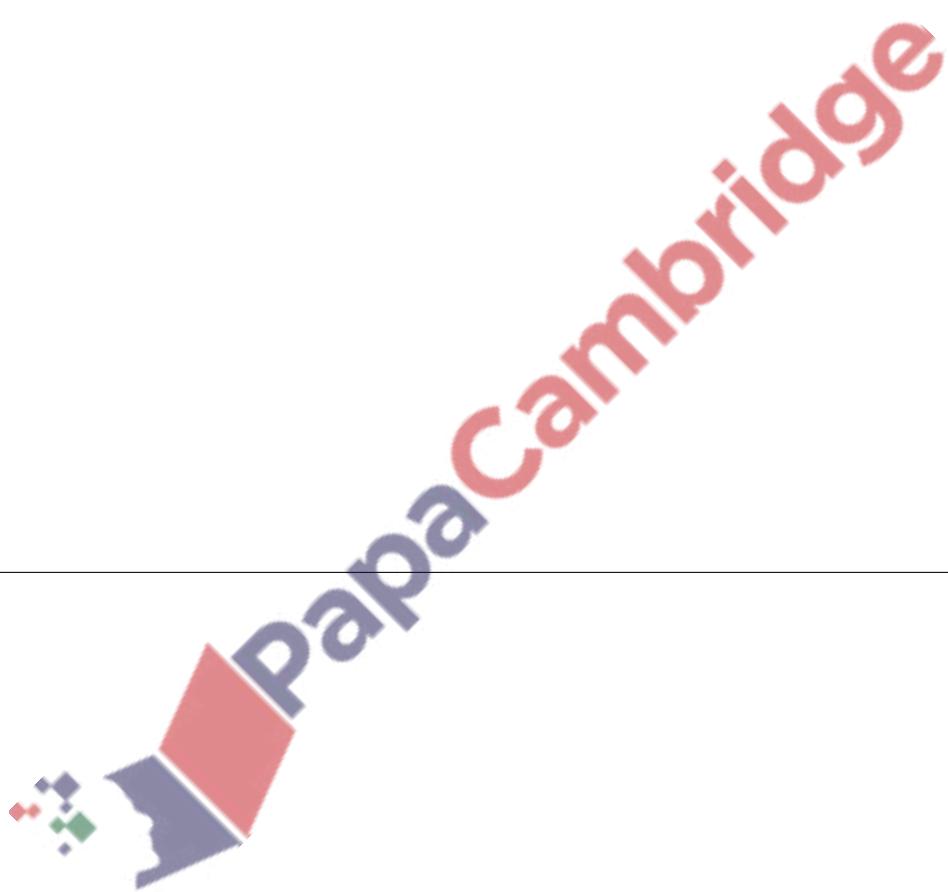


235. 9709_s16_qp_12 Q: 7

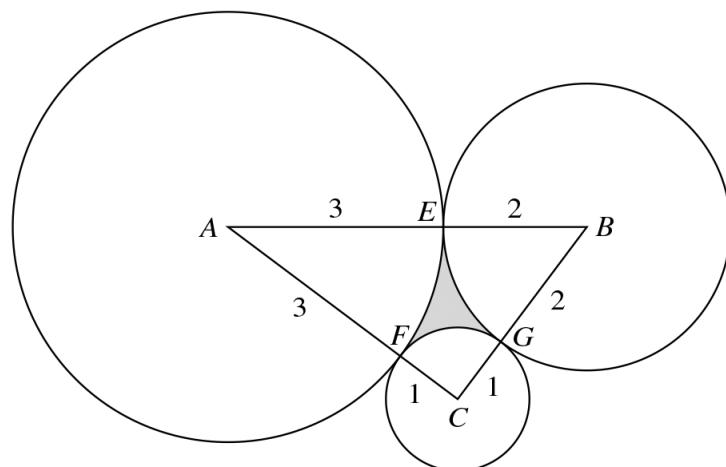
(i) Prove the identity $\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \equiv \frac{4}{\sin \theta \tan \theta}$. [4]

(ii) Hence solve, for $0^\circ < \theta < 360^\circ$, the equation

$$\sin \theta \left(\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \right) = 3. \quad [3]$$

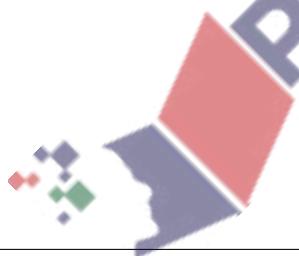


236. 9709_s16_qp_13 Q: 6



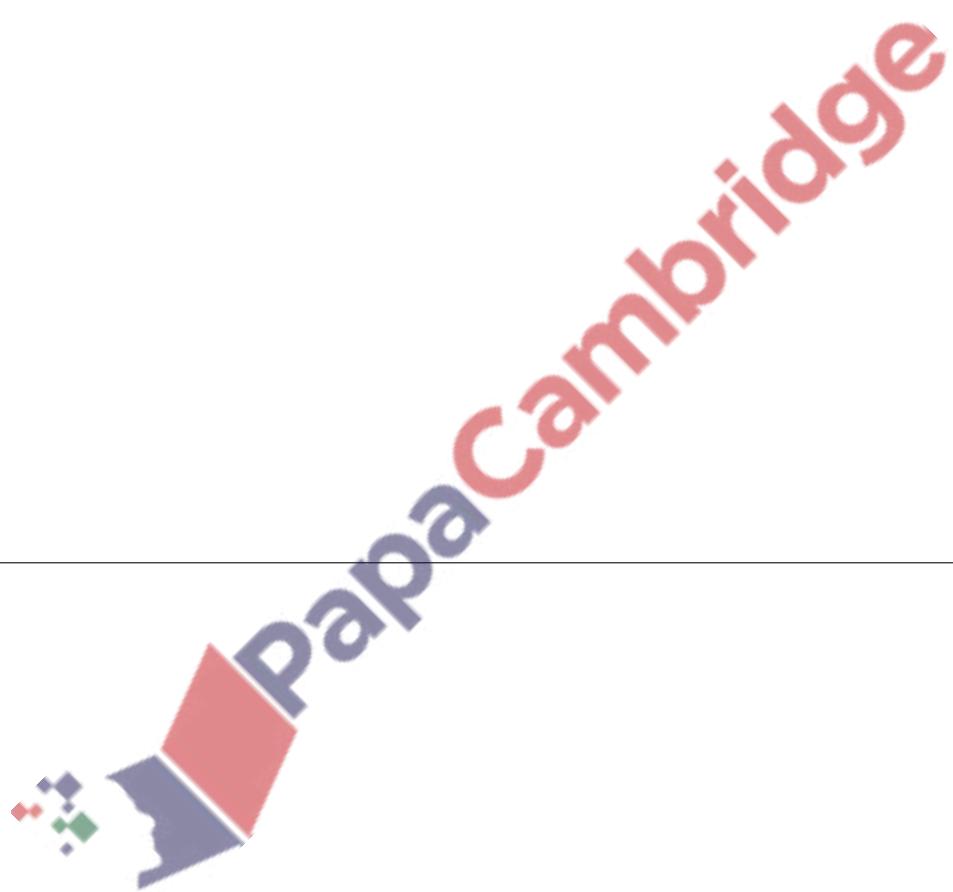
The diagram shows triangle ABC where $AB = 5 \text{ cm}$, $AC = 4 \text{ cm}$ and $BC = 3 \text{ cm}$. Three circles with centres at A , B and C have radii 3 cm , 2 cm and 1 cm respectively. The circles touch each other at points E , F and G , lying on AB , AC and BC respectively. Find the area of the shaded region EFG .

[7]

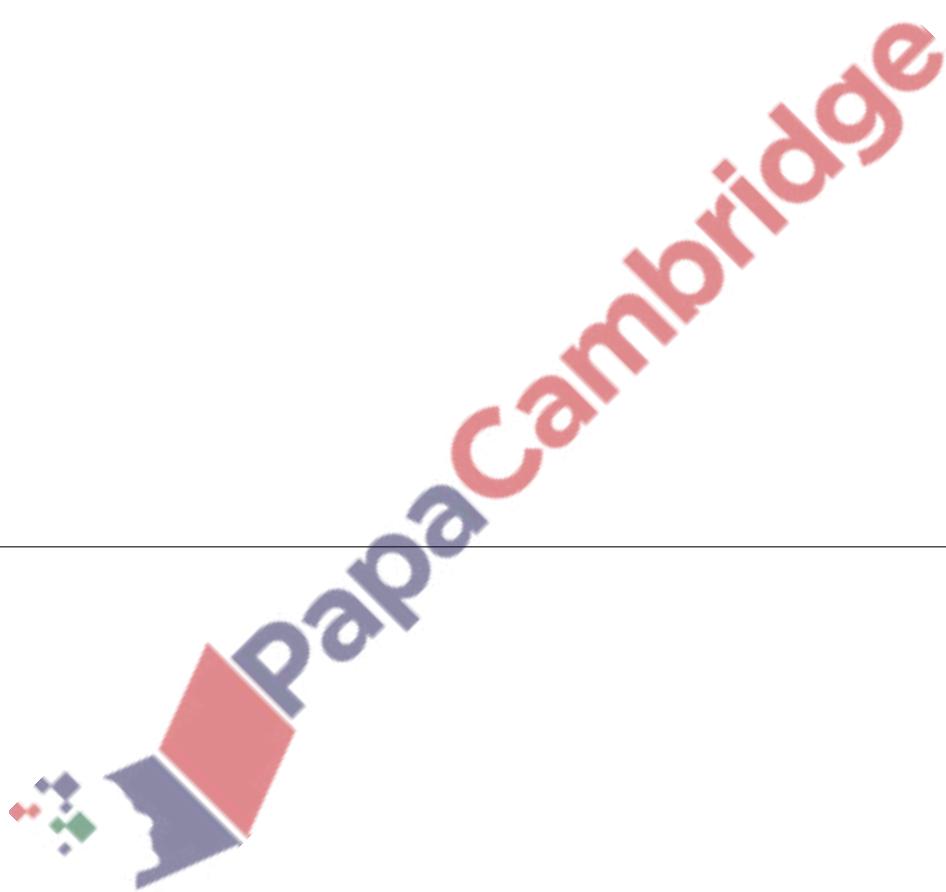


237. 9709_s16_qp_13 Q: 8

- (i) Show that $3 \sin x \tan x - \cos x + 1 = 0$ can be written as a quadratic equation in $\cos x$ and hence solve the equation $3 \sin x \tan x - \cos x + 1 = 0$ for $0 \leq x \leq \pi$. [5]
- (ii) Find the solutions to the equation $3 \sin 2x \tan 2x - \cos 2x + 1 = 0$ for $0 \leq x \leq \pi$. [3]

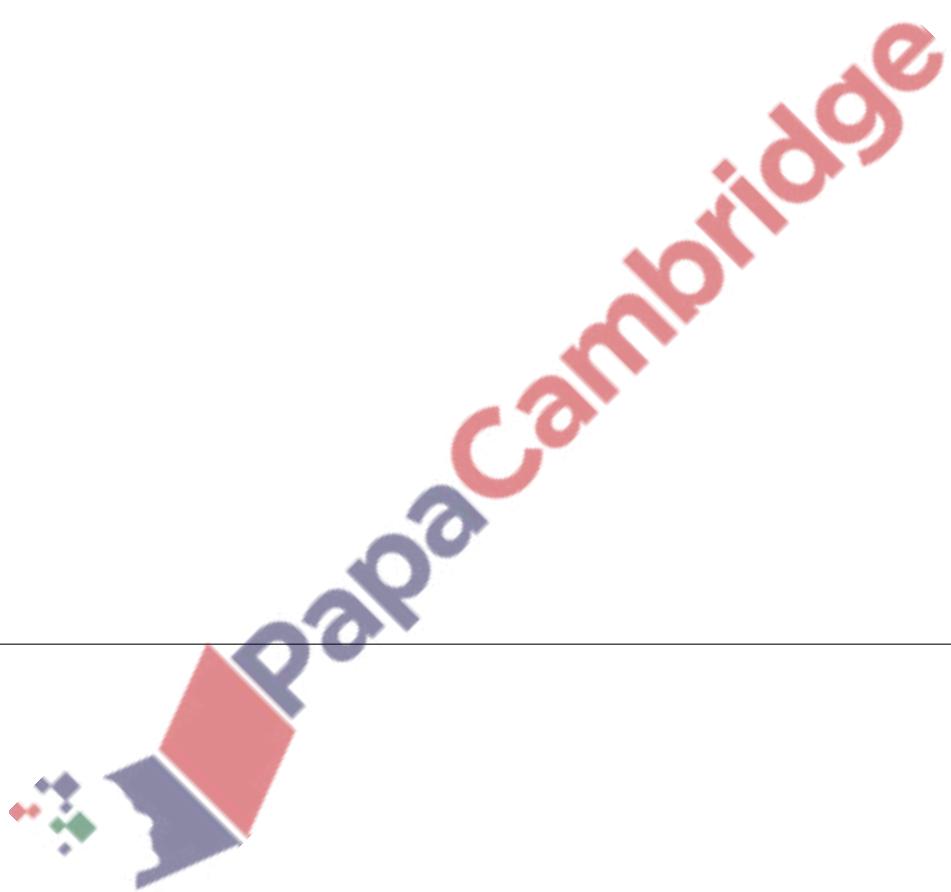


238. 9709_w16_qp_11 Q: 6

(i) Show that $\cos^4 x \equiv 1 - 2 \sin^2 x + \sin^4 x$. [1](ii) Hence, or otherwise, solve the equation $8 \sin^4 x + \cos^4 x = 2 \cos^2 x$ for $0^\circ \leq x \leq 360^\circ$. [5]

239. 9709_w16_qp_12 Q: 2

- (i) Express the equation $\sin 2x + 3 \cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]
- (ii) Hence solve the equation for $-90^\circ \leq x \leq 90^\circ$. [3]



240. 9709_w16_qp_12 Q: 10

A function f is defined by $f : x \mapsto 5 - 2 \sin 2x$ for $0 \leq x \leq \pi$.

(i) Find the range of f . [2]

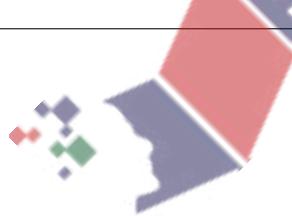
(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Solve the equation $f(x) = 6$, giving answers in terms of π . [3]

The function g is defined by $g : x \mapsto 5 - 2 \sin 2x$ for $0 \leq x \leq k$, where k is a constant.

(iv) State the largest value of k for which g has an inverse. [1]

(v) For this value of k , find an expression for $g^{-1}(x)$. [3]



241. 9709_w16_qp_13 Q: 3

Showing all necessary working, solve the equation $6 \sin^2 x - 5 \cos^2 x = 2 \sin^2 x + \cos^2 x$ for $0^\circ \leq x \leq 360^\circ$. [4]

242. 9709_s15_qp_11 Q: 1

Given that θ is an obtuse angle measured in radians and that $\sin \theta = k$, find, in terms of k , an expression for

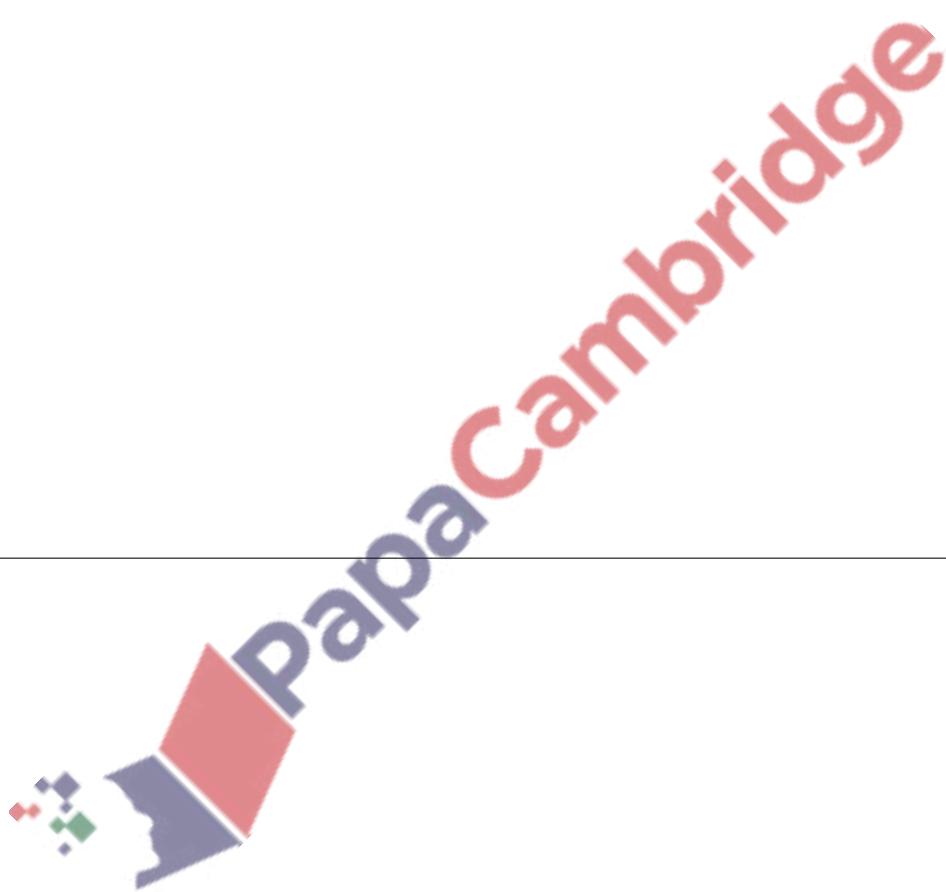
- (i) $\cos \theta$, [1]
- (ii) $\tan \theta$, [2]
- (iii) $\sin(\theta + \pi)$. [1]



243. 9709_s15_qp_11 Q: 8

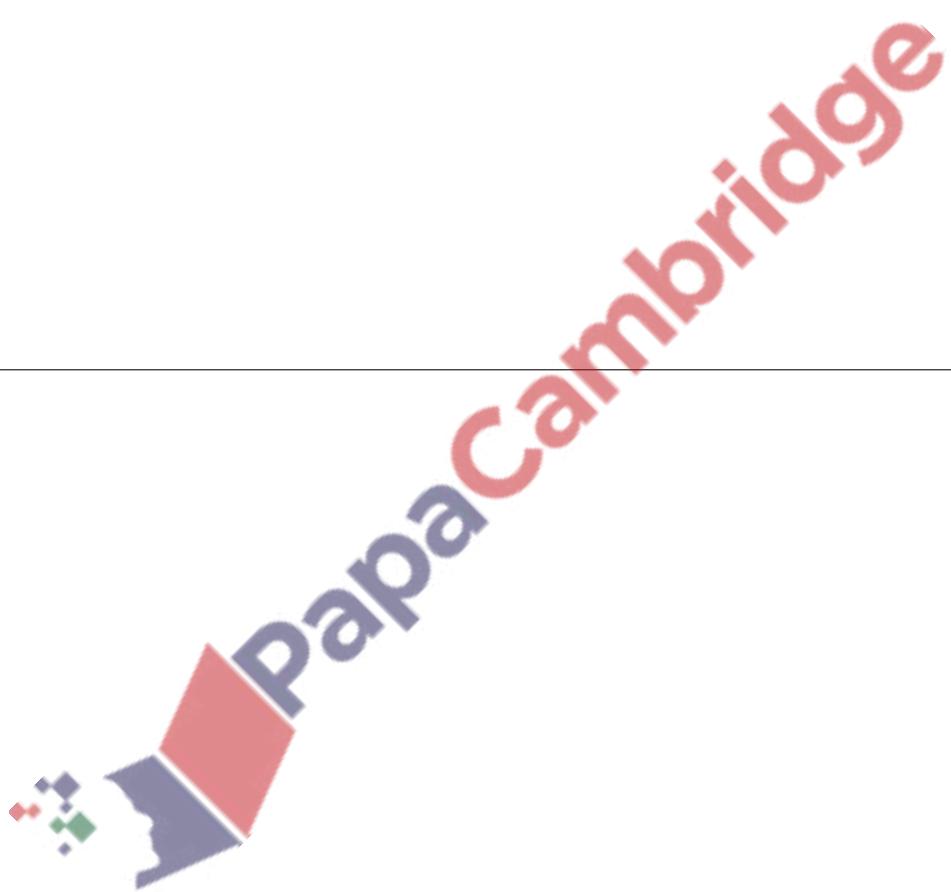
The function $f : x \mapsto 5 + 3 \cos\left(\frac{1}{2}x\right)$ is defined for $0 \leq x \leq 2\pi$.

- (i) Solve the equation $f(x) = 7$, giving your answer correct to 2 decimal places. [3]
- (ii) Sketch the graph of $y = f(x)$. [2]
- (iii) Explain why f has an inverse. [1]
- (iv) Obtain an expression for $f^{-1}(x)$. [3]



244. 9709_s15_qp_12 Q: 1

The function f is such that $f'(x) = 5 - 2x^2$ and $(3, 5)$ is a point on the curve $y = f(x)$. Find $f(x)$. [3]



245. 9709_s15_qp_12 Q: 8

- (a) The first, second and last terms in an arithmetic progression are 56, 53 and -22 respectively. Find the sum of all the terms in the progression. [4]
- (b) The first, second and third terms of a geometric progression are $2k + 6$, $2k$ and $k + 2$ respectively, where k is a positive constant.
- (i) Find the value of k . [3]
- (ii) Find the sum to infinity of the progression. [2]

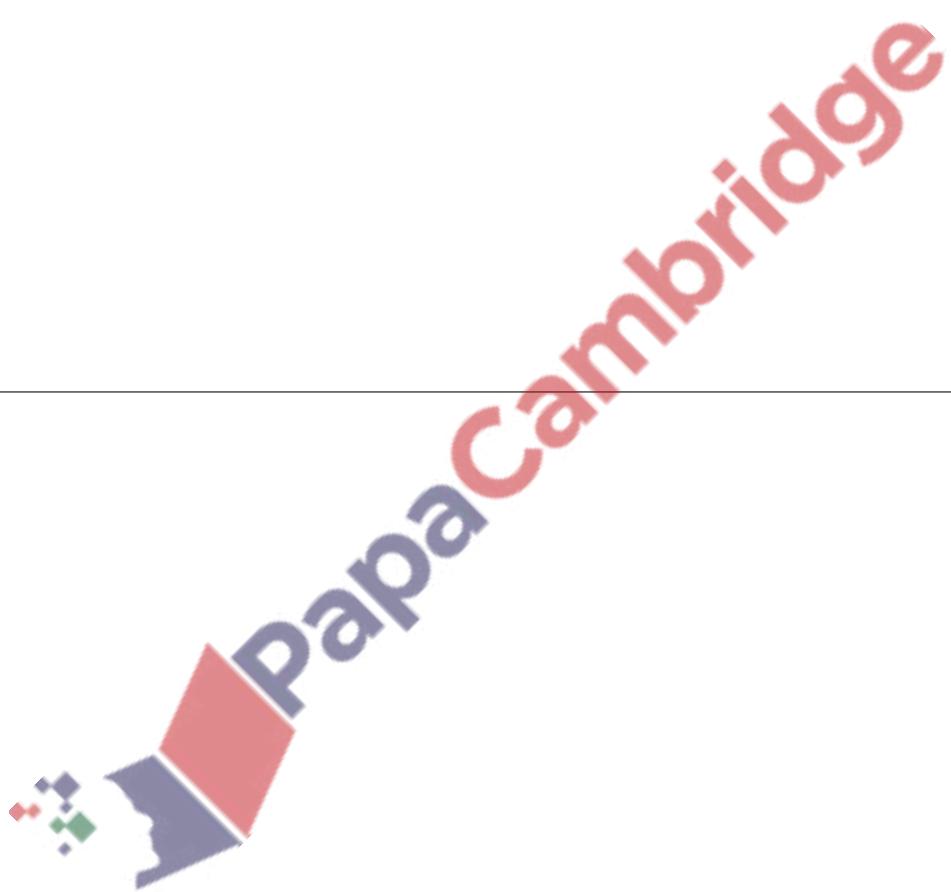
246. 9709_s15_qp_13 Q: 4

- (i) Express the equation $3 \sin \theta = \cos \theta$ in the form $\tan \theta = k$ and solve the equation for $0^\circ < \theta < 180^\circ$. [2]
- (ii) Solve the equation $3 \sin^2 2x = \cos^2 2x$ for $0^\circ < x < 180^\circ$. [4]



247. 9709_w15_qp_11_Q: 3

Solve the equation $\sin^{-1}(4x^4 + x^2) = \frac{1}{6}\pi$. [4]

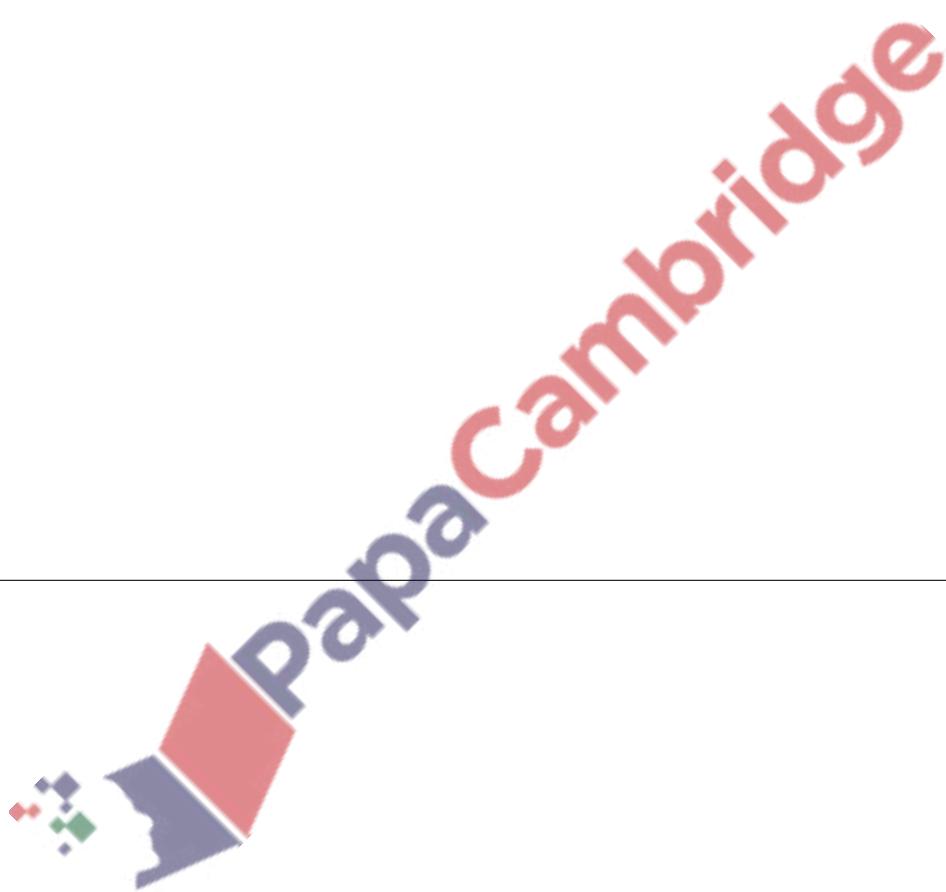


248. 9709_w15_qp_11 Q: 4

- (i) Show that the equation $\frac{4 \cos \theta}{\tan \theta} + 15 = 0$ can be expressed as

$$4 \sin^2 \theta - 15 \sin \theta - 4 = 0. \quad [3]$$

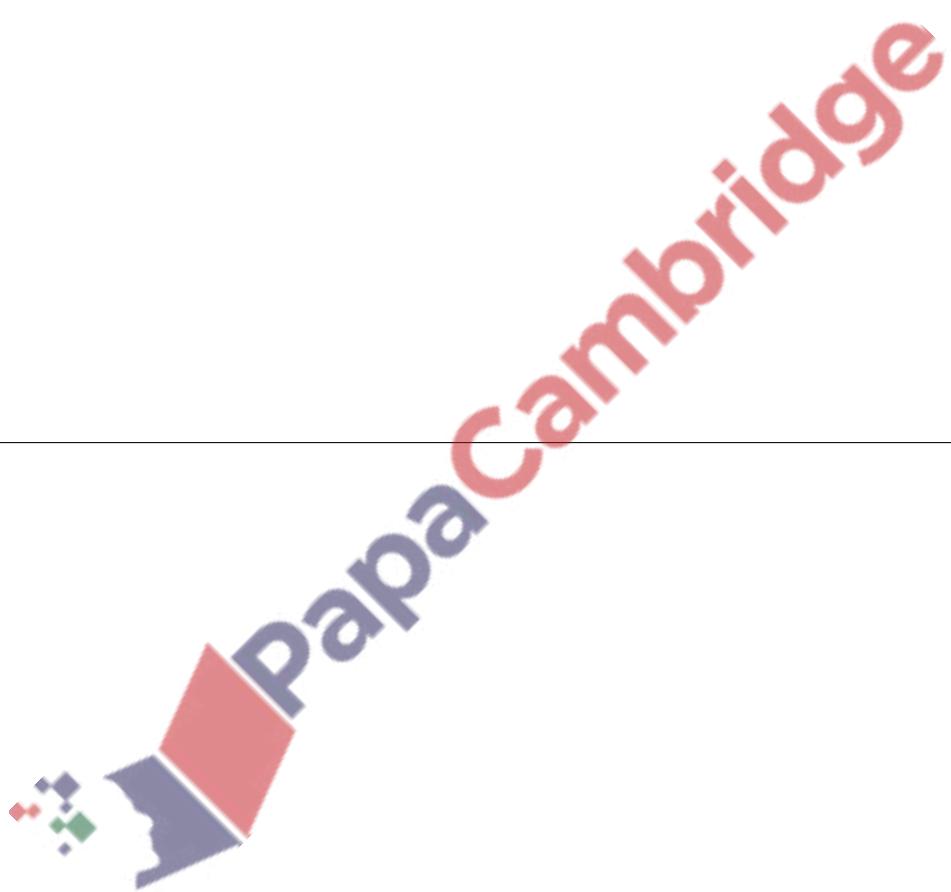
- (ii) Hence solve the equation $\frac{4 \cos \theta}{\tan \theta} + 15 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. [3]



249. 9709_w15_qp_12 Q: 4

(i) Prove the identity $\left(\frac{1}{\sin x} - \frac{1}{\tan x} \right)^2 \equiv \frac{1 - \cos x}{1 + \cos x}$. [4]

(ii) Hence solve the equation $\left(\frac{1}{\sin x} - \frac{1}{\tan x} \right)^2 = \frac{2}{5}$ for $0 \leq x \leq 2\pi$. [3]



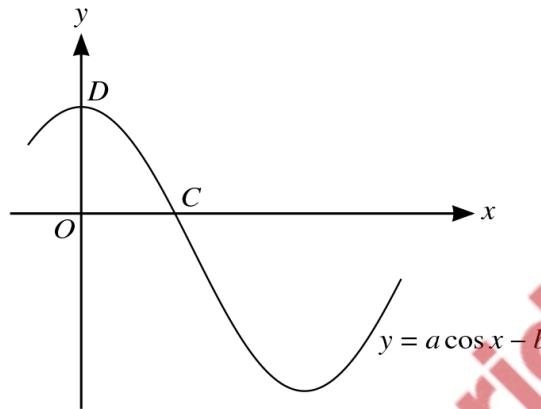
250. 9709_w15_qp_13 Q: 7

- (a) Show that the equation $\frac{1}{\cos \theta} + 3 \sin \theta \tan \theta + 4 = 0$ can be expressed as

$$3 \cos^2 \theta - 4 \cos \theta - 4 = 0,$$

and hence solve the equation $\frac{1}{\cos \theta} + 3 \sin \theta \tan \theta + 4 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. [6]

(b)



The diagram shows part of the graph of $y = a \cos x - b$, where a and b are constants. The graph crosses the x -axis at the point $C(\cos^{-1} c, 0)$ and the y -axis at the point $D(0, d)$. Find c and d in terms of a and b . [2]

